Uncertain Productivity Growth and the Choice between FDI and Export

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Uncertain Productivity Growth and the Choice between FDI and Export

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Abstract

With aggregate sales by foreign affiliates exceeding world exports, determinants of FDI patterns have received great attention, while the timing of their surge has been understudied. Recent evidence indicates that transportation costs of goods have fallen too little to explain these figures based on the *proximity-concentration* trade-off argument alone. Contextually, other changes have occurred: in particular, the uncertainty that firms bear has increased. Enriching the classical choice problem of a multinational firm with insights from the investment literature, we show that increased uncertainty along with the sizable fixed costs characterizing engagements on international markets can explain specific FDI patterns.

JEL: F17, F21, F23

Key Words: Proximity-Concentration Hypothesis, Stochastic Processes, Real Option

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1 Introduction

A prominent feature of the data in the last decades is the double digit growth of foreign direct investment (FDI) (Navaretti & Venables, 2004; UNCTAD, 2011), with aggregate sales by foreign affiliates having exceeded world exports (Raff & Ryan, 2008). To explain these patterns, the *proximity-concentration* trade-off, first presented in Brainard's (1993, 1997) seminal contributions, has become the workhorse model within Dunning's "*ownership, location, internalization - OLI*" framework. Recent advances in the trade theory have extended this basic framework to include a more involved treatment of the firm: not simply a multinational firm with two-plant operations, but also with multiple platforms (Ekholm et al., 2007), or with different efficiency levels (Helpman et al., 2004), or with more complex acquisition strategies of foreign affiliates (Nocke and Yeaple, 2007).

While these contributions have successfully explained the observed patterns of exporting and FDI strategies at any point in time, even in very narrowly defined industries, they cannot fully account for the dynamics and timing of alternative internationalization strategies (cf. Takechi, 2011; UNCTAD, 2011). According to the proximity-concentration trade-off argument, only a swing in transportation costs over time could explain the observed waves of FDI. But the available evidence generally points only to a slow, although steady, decline of air and ocean fares (Hummels, 2007). If anything, such patterns of transportation costs should imply the consolidation of exporting.

The aim of our paper is to reconcile this puzzle with the theory based on the proximity-concentration trade-off. Clearly, changes other than transportation costs can affect this trade-off, too. Neary (2009) proposes to look at regionalism and foreign acquisition as driving forces behind this change. Our paper proposes to investigate productivity evolvements and uncertainty as determinants of this change.

Indeed, recent evidence has disclosed that uncertainty which firms bear has risen: the volatility of earnings, sales, employment, and capital expenditure at the firm level have all increased in the last decades in relation to R&D investments and industry deregulations (Comin and Philippon, 2005; Comin and Mulani, 2006). What these figures imply for FDI investments remains unexplored in the literature. Yet, a lesson that clearly stems from the investment literature of the last decades is that fixed and sunk costs have important consequences for the value of action and inaction of firms in an uncertain and dynamic context (Bertola, 2010; Stokey, 2009; Dixit and Pindyck, 1994). Given that fixed costs associated with either exporting or FDI are considered sizable and partly sunk, and their role is prominent in all the recent trade literature (Das et al., 2007), it is perhaps not surprising that these costs ought to have important consequences for firms' strategies in an uncertain context.

To introduce uncertainty, we relax the common assumption in the literature that productivity is unchanged in the course of the firm's life and we assume instead that firm's productivity grows either deterministically or stochastically. This choice favors also the comparability with the static case of no productivity growth. To retain the analytical tractability of our model, we present three stylized types of uncertainty introduced in the investment literature. First, after a market entry the firm may be forced to exit the foreign market (firm's death). Second, the project the firm has planned to undertake at a future date may become suddenly unavailable (project death). Third, the firm is repetitively confronted in the course of its life with the adoption of new technologies, which may cause temporary disruption costs as well as productivity spurts (stochastic productivity).[1]

We expressly restrict ourselves to the case of exporting and FDI being perfect substitute modes: in this sense, our work complements Rob and Vettas (2003) and reflects our distinct aims. While we revise the proximity concentration framework in a dynamic and uncertain setting, they analyze conditions under which complementarity between exporting and FDI modes may emerge in a context of a stochastic non-decreasing demand. Similarly to them, and to Neary (2009), we opt for a partial equilibrium approach, in an otherwise standard monopolistic competitive setting. Although this approach disregard "crowding-out" effects, which are investigated in Head and Ries (2004), it makes possible to endogenously determine the timing of FDI, opening the perspective of explaining FDI waves within the *proximity-concentration* trade off framework.

Given the proximity-concentration trade-off assumptions, we find that market entry can be biased toward FDI in an uncertain world. Like in the static models, more productive firms

[1] cf. Holmes et al. (2008)

favor FDI to exporting because they can save on the transportation costs. The fixed cost is of less concern for them because they can amortize it better than low productive firms. If productivity is growing and firms could wait, low productive firms would become prospectively more productive and, if productivity had increased enough, their choice could lean toward FDI instead. In the real option framework, entry, although irreversible, is postponable; therefore firms can wait rather than face a "now or never" decision, as it is in the static models with a free entry condition. Provided that waiting is optimal, the firms that wait will favor FDI in prospective terms. To wait can be indeed optimal in a dynamic framework (McDonald and Siegel, 1986). This result has been a major contribution in the investment literature and remains central to our result. The diametrical behavior with firms either entering or staying out of the market implied by static models (e.g. in Helpman et al., 2004) is substituted in this dynamic setting with firms sorting along the time dimension. In an infinite time horizon with a positive productivity growth, all firms will eventually enter the market but at different points in time, opening to richer sorting patterns than those derived in a static scenario.

The remainder of this paper is organized as follows. The next section delineates the model and motivates our assumptions. Then we analyze the case of only one investment, namely exporting, to introduce the option value function that arises in the dynamic setting with irreversible fixed costs and postponable entry. Thereafter, we study the firm's choice between the export and the FDI mode in both a deterministic and an uncertain scenario. We find that the deterministic case is especially useful to build the intuition for the results that we obtain in the more complex uncertain scenario. The final section concludes.

2 The Model

A risk-neutral firm holds multiple investment opportunities, indexed by i.[2] Once executed, investment i is *irreversible* and yields a growing periodical cash-flow $\pi(i,t)$ in each instant of time t. The cost of the investment $I(i)$ is constant and incurred by the firm in the period in which the investment is executed, and thereafter is sunk.

[2] In the seminal paper by McDonald and Siegel (1986) which analyzes the optimal timing of an irreversible project the firm possesses one investment choice.

The irreversible nature of the investment combined with an endogenous time of execution, is what sets the approach presented here apart from the traditional monopolistic competitive model commonly used in international economics (see e.g. Helpman, Melitz and Yeaple, 2004). In this model, a firm can only decide whether to enter in a market in given time, and not when to enter the market. The firm is confronted with a "*now or never*" choice, and the optimal action is pursued if the benefits of action outweigh its costs.[3] In the real option approach presented here, however, a firm has in addition the option to "*wait*", which yields economic benefits that have to be factored in the firm's decision to undertake the investment.

With an option to wait, the point in time in which the firm decides upon investment opportunity i can be different from the period in which the firm executes it. Without loss of generality, at $t = 0$ (the initial period) the firm decides when (and if) to undertake the investment at a future date $T(i) \geq 0$. Therefore, $T(i)$ represents both the execution period of the investment and the waiting time of the firm: if the firm opts for executing the investment right from the initial period (i.e. $T(i) = 0$), waiting must be suboptimal.

Specifically, the investment opportunity i the firm considers is entry in a foreign market. The demand in the foreign market for the firm's output X is

$$p(t) = Z(t)X(t)^{\nu-1}, \quad 0 \leq \nu \leq 1 \tag{1}$$

where ν is the degree of competition in the market. For $\nu = 1$, the demand curve is perfectly elastic (i.e. the market is perfectly competitive), whereas as $\nu \to 0$ the firm's monopoly power increases; $Z(t)$ represents a shift factor, including for instance the destination country's income.[4]

To produce output the firm uses only labor: let the technology be

$$X(t) = \phi(t)L(t), \qquad \phi(t) > 0, \tag{2}$$

[3] In the option literature the "now or never choice" is often referred to as a"take or leave" option (cf. Bertola, 2010)

[4] Such a demand for a variety ω in the variety set Ω can be derived from a CES utility function $U = \left(\int_{\omega\in\Omega} X(t,\omega)^\rho d\omega\right)^{\frac{1}{\rho}}$, where $\eta = \frac{1}{1-\rho} > 1$ is the elasticity of substitution. The inverse mark-up is then $\nu = \frac{\eta-1}{\eta}$.

where $\phi(t)$ is the firm's productivity.[5] We are implicitly assuming a perfect technology transfer as the firm is equally productive regardless of the type of investment undertaken.[6] The transportation cost to ship production abroad are investment specific and of the iceberg type, so that $\tau(i,t) > 1$ units of output have to be shipped for one unit of output to be delivered at a foreign destination.

Finally, let $w(i,t)$ be the factor price of labor under investment i. Solving the firm's profit maximization problem, the highest periodical cash-flow associated with investment i is

$$\pi(i,t) = M(i,t)\phi(t)^{\kappa}, \tag{3}$$

where $M(i,t) = Z(t)^{\frac{1}{1-\nu}} \left(\frac{\nu}{w(i,t)\tau(i,t)^{\frac{1}{\cdot}}} \right)^{\kappa} (1-\nu)$ and $\kappa = \frac{\nu}{1-\nu}$. $\pi(i,t)$ is increasing in the level of productivity $\phi(t)$, and convex for $\kappa > 1$. The lower the input factor prices or the transportation costs, the lower the firm's variable costs and therefore the higher $M(i)$ and the profits.

As stated above the cash-flow is evolving over time: specifically, we shall assume motion in productivity to compare our results with those obtainable with the monopolistic competitive model with heterogeneous firms. However, our model is more general and can encompass, also simultaneously, other sources of motion of the firm's cash-flow like growing foreign demand or price changes.[7]

We decompose our analysis into two parts - deterministic and uncertain cases - which allows us to derive new results that already arise in a simple deterministic dynamic framework if fixed costs are irreversible. Such an approach is wise as it briefly illustrates the real-option effect within a deterministic environment while unfamiliar readers can easily understand the mechanism. Therefore, unlike McDonald and Siegel, we first rule out cash-flow uncertainty

[5] ϕ could also be re-interpreted as a quality index of the good produced.

[6] It is easy to extend the model by introducing a concave production function in (2) such as $X(t) = \phi(t)L(t)^{\theta}$ with $0 < \theta < 1$. However, with reference to the monopolistic competition models that generally assume constant returns to scale technologies (e.g. Helpman et al., 2004) we consider only cases with $\theta = 1$.

[7] E.g. if the profit of a firm is the product of several stochastic variables, but each variable's motion is individually described by a Geometric Brownian motion, then the profit itself behaves as a Geometric Brownian motion. The resulting drift and diffusion parameters are linear combinations of the corresponding single process parameters (Bertola, 1998).

and present the case of deterministic productivity growth - our benchmark case - to which we compare the effects of different kinds of uncertainty, afterwards. The law of motion of productivity is therefore

$$d\phi(t) = \alpha\phi(t)dt, \tag{4}$$

where α represents the productivity growth rate. Consistently with our goals to be as close to the monopolistic competitive settings as viable, we analyze production technologies with constant returns to scale, implying $\kappa > 1$. Furthermore, as all variables but productivity are constant over time, $M(i,t) = M(i)$. To simplify the notation, let us normalize the initial level of productivity to $\phi(0) = \phi$.

2.1 A single investment: market entry

As cash-flows will begin to accrue first after entry at time $T(i)$ at the level $\pi(i, T(i))$ and will continue to grow at rate $a = \alpha k$, the cash-flow associated to investment i in any given period of time is[8]

$$\pi(i,t) = \begin{cases} 0, & \text{for } t < T(i) \\ \pi(i,0)e^{at}, & \text{for } t \geq T(i). \end{cases} \tag{5}$$

The discounted value of these stream of profits net of the entry cost $I(i)$ will constitute the net value of market entry: letting e^{-rt} denote the discount factor, the net present value of the investment i executed at $t = T$ as of time 0 is

$$\begin{aligned} V_0(i,\phi,T(i)) &= \int_0^\infty \pi(i,t)e^{-rt}dt - I(i)e^{-rT(i)} \\ &= \int_{T(i)}^\infty \pi(i,0)e^{at}e^{-rt}dt - I(i)e^{-rT(i)} \\ &= e^{-rT(i)}\left(\frac{\pi(i,T(i))}{r-a} - I(i)\right). \end{aligned} \tag{6}$$

[8] Given (4), the adjusted growth rate for convex cash-flows results as $a = \frac{d\phi^\kappa}{\phi}\left(\frac{1}{dt}\right) = \alpha\kappa$.

We assume $r > a$ for a well defined problem. Clearly, a firm confronted at $t = 0$ with a "*now or never*" option of entering a market at $T(i)$ should adopt the investment i only if its value exceeds its costs, or if:

$$V_0(i, \phi, T(i)) \geq 0 \Rightarrow \quad \pi(i, T(i)) \geq (r - a)I(i). \tag{7}$$

Equation (7), known as the *Marshallian rule*, just ensures that entry will yield at least zero net value. But, intuitively, the firm ought to reach even a higher value of investment if the firm has in addition the strategic choice about the time of entry.

With an *option to wait*, how long a firm should be opportunely delaying entry is a balancing act between the costs and the benefits associated with waiting. On the one hand, the firm foregoes profit opportunities as it delays entry. On the other hand, as time passes, cash-flows will start from a higher value (if $a > 0$) and simultaneously capital gains on the saved entry cost $I(i)$ will yield a financial dividend. If the financial benefits outweigh the lost stream of cash-flows, it is optimal to delay the payment of $I(i)$ and to enter the market when the cash-flow's present discounted value has grown larger.

Equation (6) illustrates this balancing act analytically. Accordingly, the waiting time $T(i)$ affects the net present value of investment i in two opposite ways: the first term, the discount factor $e^{-rT(i)}$, declines with $T(i)$ (if $r > 0$); the second term increases with $T(i)$ (if $a > 0$), but its evolution is slower than exponential as forgone cash-flows grow over time but are reduced by $I(i)$. Therefore, equation (6) represents a trade-off between the marginal gains that arise from financial benefits through $I(i)$ if the firm stays out of the market, and the potential periodical profits $\pi(i, T(i))$. The longer the firm waits the smaller the financial benefits become, due to increasing forgone profits up to the point at which periodical profits are larger then the marginal financial benefits through $I(i)$. This trade-off hinges on a being positive and market entry is optimal only when the net value of the investment is maximized. Formally, we can derive the optimal waiting time as:

$$O(i, \phi, a, I(i), r) = \max_{T(i) \geq 0} e^{-rT(i)} \left(\frac{\pi(i, T(i))}{r - a} - I(i) \right). \tag{8}$$

Implicitly, the optimal entry rule for a firm that holds the option to wait is

$$\pi(i, T^*(i,\phi)) \geq rI(i) \quad (\text{with } = \text{ if } T(i,\phi)^* > 0) \tag{9}$$

where $T^*(i,\phi)$ denotes the optimal entry time. The trigger rule for market entry is essentially a non-arbitrage condition and states that the cash-flow should be as large as the user cost of capital when the investment is executed. Otherwise, the firm can do better by delaying market entry and keeping the sum of $I(i)$ invested in financial assets yielding a dividend of $rI(i)$ till the payoff at entry of the real investment at least equates this dividend.

The optimal condition expressed in equation (9) is known as the *Jorgensonian rule* from Jorgenson (1963) and is more restrictive than the *Marshallian rule* expressed in (7), as the cash-flow that triggers market entry in (9) is larger than its counterpart in (7), provided that $a > 0$; and they are equal only for $a = 0$. Therefore, in a dynamic framework the Marshallian rule is only a necessary but not sufficient condition for the optimality of the investment.

Clearly, the trigger rule for market entry (9) defines implicitly the optimal entry time. Market entry in the initial period (immediate entry) can only be optimal if the profit in the initial period covers the user cost of capital, that is if $\pi(i,0) = rI(i)$. Using the expression for periodical profits (3), the threshold level of initial productivity above which immediate market entry (i.e. no waiting) is optimal is

$$\phi^*(i) = \sqrt[\kappa]{\frac{rI(i)}{M(i)}}. \tag{10}$$

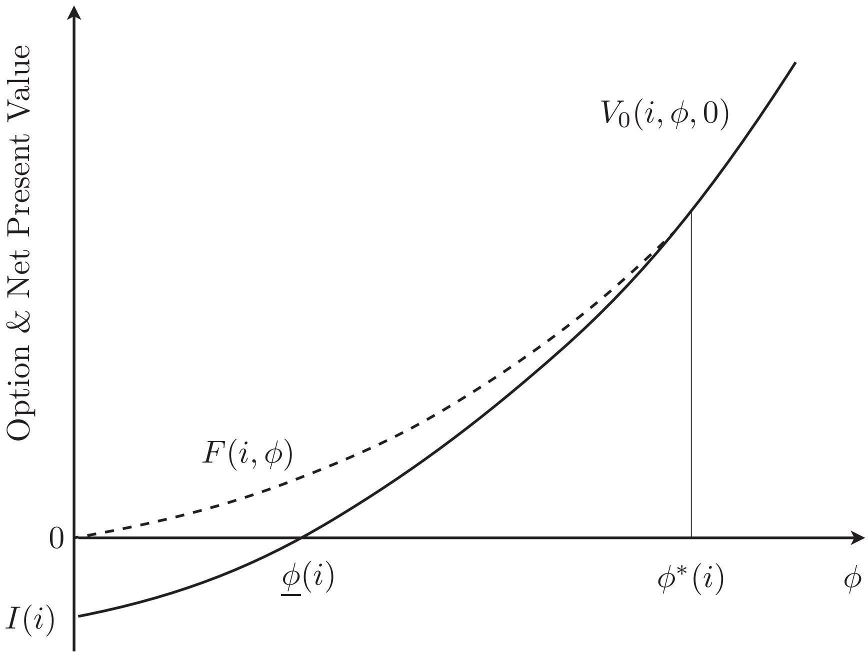
For initial productivity levels below this threshold, the firm will find it optimal to wait till $T^*(i,\phi)$. Solving explicitly for the optimal entry time and substituting it into (8), the firm's highest net value of investment for any given level and growth rate of productivity is[9]

$$O(i,\phi,a,I(i),r) = \begin{cases} A(i)\phi^\beta \equiv F(i,\phi) & \phi < \phi^*(i) \\ V_0(i,\phi,0) & \phi \geq \phi^*(i)\ , \end{cases} \tag{11}$$

[9] The explicit solution to the optimal entry time is $T^* = \begin{cases} 0 & \text{if } \phi \geq \phi(i)^* \\ \frac{1}{a} ln\left(\frac{rI}{\pi(i,0)}\right) & \text{otherwise.} \end{cases}$

where $A(i) = \frac{a}{r-a} I(i) \left(\frac{M(i)}{rI(i)}\right)^{\frac{\beta}{\kappa}}$, $\beta = \frac{r}{\alpha}$. In figure 1, we depict it as the envelope of two curves: the dashed curve $F(i,\phi)$ in the segment $(0, \phi^*(i))$, and the solid curve $V_0(i,\phi,0)$ in the segment $(\phi^*(i), \infty)$. The curve $V_0(i,\phi,0)$ is the net value of investment i executed

Figure 1: Value Functions of an Investment



Note: The solid line plots the discounted value of an investment i at time $T = 0$. The dashed line represents the respective option function $F(i,\phi)$. At a productivity level $\phi^*(i)$ the two functions are tangent. For any productivity level larger $\phi^*(i)$ the investment is immediately adopted, while productivity levels in the range $(\underline{\phi}(i), \phi^*(i))$ will lead to an investment postponement. $\alpha > 0 \wedge r > \alpha$.

immediately and therefore represents the value of a "*now or never*" investment opportunity with $T = 0$. This value is negative for low productivity levels as an unproductive firm cannot generate a sufficient stream of cash-flows to recoup the fixed cost of the investment. The curve $F(i,\phi)$ is the net value of investment i executed at $T^*(i,\phi)$ and evaluated in $t = 0$ and therefore, includes the value of waiting. Since waiting yields financial benefits resulting from delaying the investment's fixed cost payment, this option value is always non-negative for any level of productivity; it starts at the origin and is increasing and convex in productivity. The benefit and the costs of waiting exactly balance only at a productivity level $\phi^*(i)$. Short of this level, delaying entry is beneficial, but passed this level the financial benefits associated with delaying the fixed costs of investment do not compensate for the foregone profits from entry and therefore, waiting is disregarded.

This is shown in figure 1 with the $F(i,\phi)$ curve being above the $V_0(i,\phi,0)$ curve for any productivity level below $\phi^*(i)$ and hence visualizing the value of waiting as a positive distance between these two functions.

The firm with productivity $\phi^*(i)$ is indifferent between immediate entry and remaining out of the market: firms with higher productivity enter the market immediately and firms with lower productivity rather wait and execute the investment at future date. How long they wait, depends both on the level of initial productivity ϕ and on its growth rate α, as implied in equation (9) (see also footnote 9).

Compare these sorting patterns with those emerging if firms only hold a "*now or never*" investment opportunity without an option to wait. In such a case, firms with productivity levels inferior to $\underline{\phi}(i)$ stay out of the market while firms with productivity levels above this threshold enter the market immediately. While the implied sorting patterns in the two cases are similar for productivity levels above $\phi^*(i)$ where a firm immediately enters the market, they are different for productivity levels below this threshold. Firms that would either enter immediately or stay out of the market when they just hold a "*now or never*" option, delay entry if they hold an option to wait. And firms that would enter from $t = 0$ with a "*now or never*" choice, delay entry in spite of the positive net value of the investment and the *Marshallian rule* being fulfilled. The marginal firm with productivity $\underline{\phi}(i)$ has exactly zero net value of investment, but, in accordance with the *Marshallian rule*, entry is the optimal behavior. However, the same firm that holds also the option to invest later, finds it optimal to postpone investment. If the opportunity cost of raising funds $I(i)$ is $r > 0$, delaying the payment of the fixed costs gives a yearly yield of $rI(i)$. As derived in (9), according to the *Jorgensonian rule* the firm should postpone the investment until the annual profit from entry guarantees at least the same payoff. To reach this level of profit the firm will have to wait $T^*(i,\underline{\phi}(i))$. This strategy results in a higher net value of investment $F(i,\underline{\phi}(i)) > 0$.

Finally, it is interesting to analyze the effects of a change in the growth rate of productivity α, implying a higher rate of profit growth a. While this change increases the net value of the investment for all firms, it does not change the current profit and therefore the productivity threshold $\phi(i)^*$ required for immediate market entry as one can deduce from equation (10).

Therefore, firms to the right of $\phi^*(i)$ keep entering the market at $t = 0$ and firms with a lower level of productivity keep postponing entry, but wait a lower number of periods as $T^*(i, \phi)$ reduces. Since $rI(i)$ is constant and profits grow faster, firms will need less time to reach the profit that triggers entry as defined in equation (9).

In a static scenario, ($\alpha = 0$), waiting is never beneficial as the profitability of the investment in the future is unchanged. Delaying the payment of the fixed costs still accrues financial benefits, but the stream of future profits from entry is not growing larger meanwhile. Therefore, if entry today is not rewarding, so it will not be tomorrow. The option value $F(i, \phi)$ is indeed null, precluding the optimality of waiting, and $F(i, \phi)$ in (11) coincides in figure 1 with the horizontal axis till $\underline{\phi}(i)$ and with $V_0(i, \phi, 0)$ afterwards.

It is therefore the combination of three elements that make waiting worth: two of them, the irreversibility of the investment and the postponement of the the investment, are related to the nature of the investment; the last one, the growth rate of the cash-flow, is related to the dynamic of the investment.

At this point it is important to emphasize that the diametrical behavior with firms either entering or staying out of the market implied by a static model is substituted in our dynamic framework with firms sorting along the time dimension. In an infinite time horizon with $a > 0$ all firms will enter the market but at different points in time. We show in the following that such a relaxation of the entry choice has important implications for the sorting pattern into different entry modes i of firms with different productivity levels.

2.2 Multiple Investments: Export and FDI

Consider now a firm that has to decide whether to enter the foreign market by exporting its good from an existing location ($i = E$), or serving the market locally by opening a foreign affiliate ($i = F$). Since the seminal paper by Brainard (1993), the choice between export and green field FDI has been explained by the argument based on the "proximity-concentration" trade-off. As unanimously accepted in the trade literature, both export and FDI activities are characterized by high fixed costs which are, at least partly, sunk. The aim of this section is to analyze how the choice between these two modes of firm's internationalization is affected by the dynamics of productivity and the possibility of investment postponement. While the *Real Option* approach typically focuses on the timing of the investment, we focus here on the type of the entry mode the firm makes.

As for the case of the single investment, the firm has to decide at $t = 0$ if and when to enter the foreign market. Additionally, it has to determine the mode of market entry. The two modes are mutually exclusive: the firm can neither serve the market by means of both modes, nor can it switch from one to the other in the course of time.[10]

The proximity concentration trade-off postulates that exporting economizes on the fixed costs of the investment by concentrating production in one site, while FDI economizes on the transportation costs by serving the market locally. We therefore assume that $I(F) > I(E)$ and $M(F) > M(E)$.[11]

Clearly, the firms will choose the strategy associated with the highest possible net value of the investment, $\max\{O(E,\cdot), O(F,\cdot)\}$. While the strategy with the highest reward will typically depend on the level of initial productivity, it is possible to identify conditions under which the FDI strategy will dominate the export strategy for all levels of productivity. This dominance is ascribable to the cost assumptions at the base of the proximity concentration trade-off.

Depending on the parameters of the model, the order relation of the two initial productivity cutoffs for market entry, $\phi^*(E)$ and $\phi^*(F)$, is not unique. Nevertheless, the interesting cases arise when $\phi^*(E) < \phi^*(F)$, implying

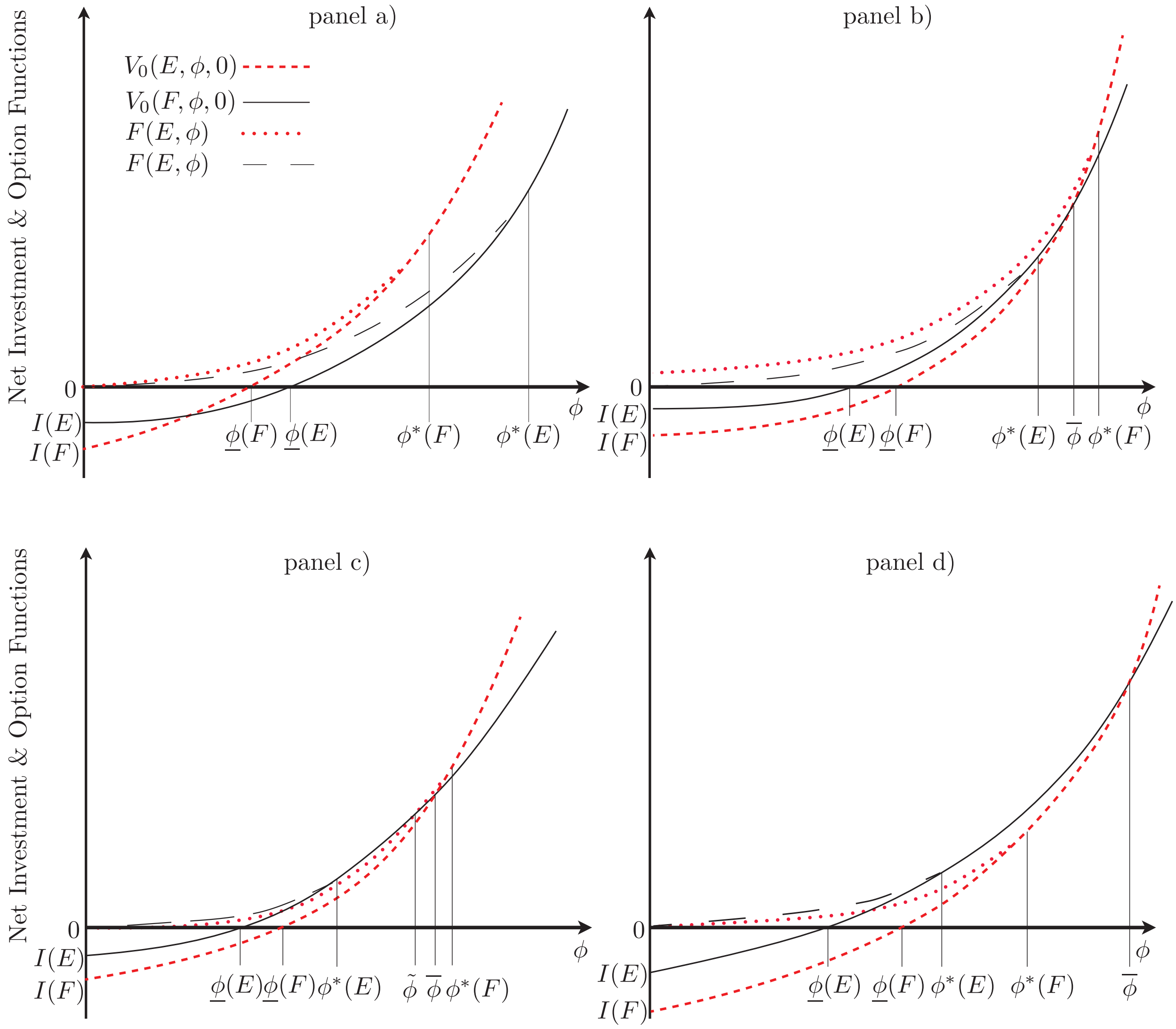
$$\frac{M(E)}{M(F)} > \frac{I(E)}{I(F)}. \tag{12}$$

Otherwise, as in panel a) — figure 2 — the two net value functions $V_0(E, \phi, 0)$ and $V_0(F, \phi, 0)$ cross each other in quadrant IV, implying that the FDI strategy would be also dominant on

[10] This assumption can be relaxed and additionally switching behavior can be considered, which would necessitate numerical simulations. As we focus only on first time market entry decisions, we omit this extension.

[11] Assuming that $\tau(F,t) = 1$, the latter assumption is equivalent to assume $w(E,t)\tau(E,t) > w(F,t)$.

Figure 2: Possible sorting patterns in the presence of growth



Note: In all four panels the solid and dashed black curves respectively represent the net present value and the option function of the export entry mode. Equivalently, the red dashed and dotted curves depict the FDI mode's net present value and option function, respectively. In panel a) and b) FDI is the optimal market entry mode independently of the firm's contemporaneous productivity level. Panel c) and d) present cases in which the optimal entry mode hinges on the firm's productivity level. In all four panels the cost constellations of the FDI and export mode comply with the proximity-concentration trade-off cost assumptions.

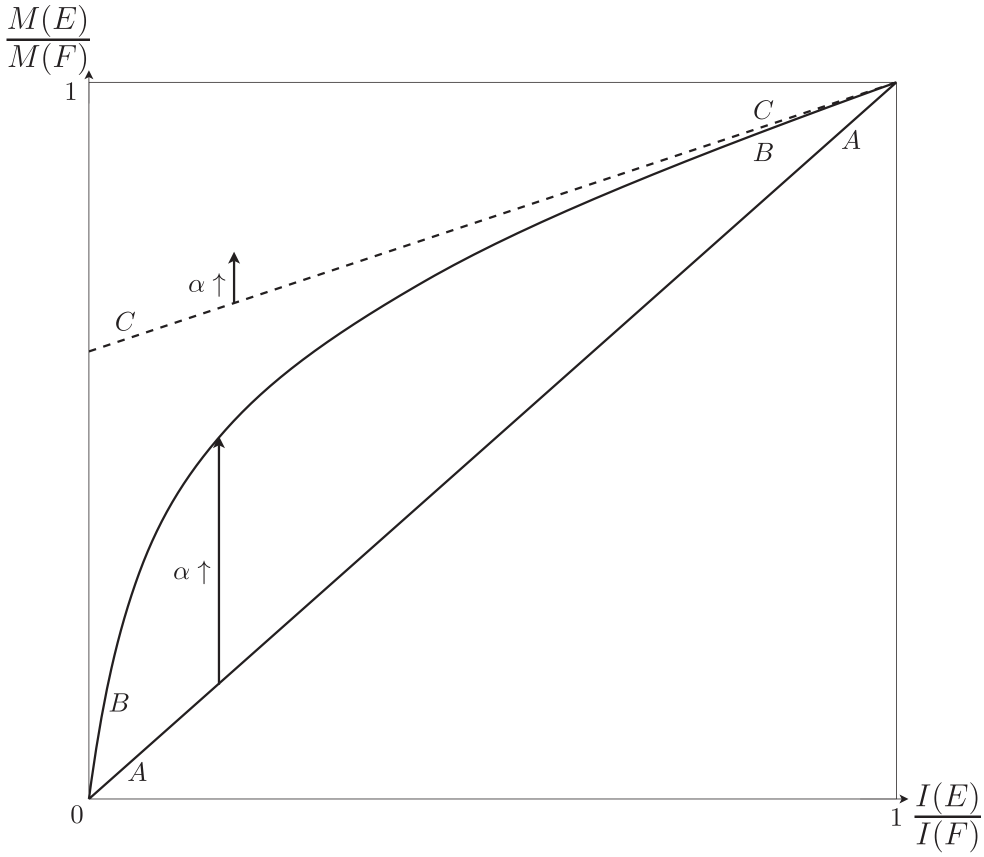
the basis of the *Marshallian rule* alone. More generally, we can state that as long as,

$$F(F, \phi^*(E)) > F(E, \phi^*(E)) \quad \Leftrightarrow \quad \frac{M(E)}{M(F)} < \frac{I(E)}{I(F)} \left[\frac{I(E)}{I(F)}\right]^{-\frac{\kappa}{\beta}} \tag{13}$$

and

$$V(F, \phi^*(F), 0) > V(E, \phi^*(F), 0) \quad \Leftrightarrow \quad \frac{M(E)}{M(F)} < \frac{a}{r} + \frac{r-a}{r}\frac{I(E)}{I(F)}, \tag{14}$$

Figure 3: The proximity-concentration relative cost space



Note: This box depicts a firm's possible relative cost constellations between an export and FDI market entry strategy which complies with the proximity-concentration trade-off cost assumptions. The horizontal axis captures the relative fixed costs, while the vertical axes measures the relative variabel costs of the respective two market entry strategies. For any cost constellation below the BB curve a firm will chose FDI as the optimal entry mode. Above the BB curve, the optimal entry mode depends on the firm's contemporaneous productivity level.

FDI will be the dominant strategy for all firms, regardless of the level of productivity.

We graph the three conditions (12) to (14) in figure 3 — where we depict the relative fixed cost on the horizontal axis and $M(E)/M(F)$, the relative "profitability", on the vertical axis to have a square box of length 1. Each curve in the box is the locus of points where each condition holds with equality: the diagonal curve labeled AA depicts condition (12), curve BB condition (13) and curve CC condition (14). As the second condition implies the third condition, the FDI dominance region is all the area below the parable (curve BB).[12] However, as explained above, the lower triangle below the diagonal line AA represents all parameters constellations where exporting is never optimal and therefore it is of minor interest for a comparative statics analysis. Above the diagonal line (curve AA) and below the parable (curve BB), FDI results dominant in our framework, but not in the static monopolistic competitive model where sorting of firms into the different activities would depend on firm's productivity.

This case is depicted in panel b) — figure 2—: considering only the net value curves for immediate entry, the *Marshallian rule* would imply that firms with productivity below $\underline{\phi}(E)$ are domestic firms, those with productivity in the range $[\underline{\phi}(E), \overline{\phi}]$ export and finally, firms with productivity above $\overline{\phi}$ perform FDI, similarly to the sorting patterns arising in Helpman et al. (2004). The sorting patterns implied by the *Jorgensonian rule* look differently as all firms perform FDI. While firms with productivity above $\phi^*(F)$ invest immediately, those with productivity below this threshold wait $T^*(F, \phi)$ periods.

Therefore, firms with a high productivity, above $\phi^*(F)$, behave identically regardless of the type of option they hold into and simply invest in greenfield FDI. However, holding the option to wait affects the behavior of the firms that are not so productive because it opens an alternative that is not available with the "*now or never*" option. The alternative to market entry when holding a now or never option is just remaining out of the market and earning nothing. Then, a non negative net value of the investment is a superior choice that justifies the investment. But with the option to "*wait*", the alternative is delaying the fixed cost of the investment and earning the financial dividend $rI(i)$. Then, investing is worth only when the dividend from the real investment can top this amount. While waiting for this moment, productivity improves and makes the FDI investment more attractive than exporting. Or stated differently, transportation costs or the relative fixed costs $I(E)/I(F)$ should reduce for the export mode to maintain its appeal.

Above the parable BB in our box, the choice between exporting and FDI depends also

[12] Both curves BB and CC are monotonically increasing between 0 and 1 and are equal to 1 in 1, but in 0 curve BB is null and curve CC is a/r. The slope of curve CC is constant and equal to $(r-a)/r$, while the slope of curve CC is $[I(E)/I(F)]^{-a/r}(r-a)/r$. Therefore the slope of the two curves is the same in 1, but for any point on the horizontal axis below 1, the slope of curve CC is higher. All together, these conditions imply that curve CC lies always above curve BB.

in our framework on firm's productivity. Panel c) — figure 2 — is indicative of the sorting patterns that emerge between the parable (curve BB) and the dashed line (curve CC), and panel d) — figure 2 — exemplifies the sorting patterns emerging above curve CC. The sorting patterns in the upper part of the box are similar to the one emerging in Helpman et al. (2004). Firms with productivity above $\overline{\phi}$ undertake FDI, while firms with intermediate levels of productivity in the range $[\phi^*(E), \overline{\phi}]$ are exporters (from the initial period). The difference is that firms with initial productivity below $\phi^*(E)$ will, in our framework, become exporters in $T^*(E)$ periods, while in the static framework they either enter immediately, or remain domestic firms only.

Compared with the static monopolistic competition models (cf. Helpman et al., 2004) the sorting pattern between curve BB and CC depicted in panel c) — figure 2 — are more rich, in that waiting arises in equilibrium for both strategies, a feature that is specific only to the Real Option approach. Firms with productivity below $\phi^*(E)$ and in the range $[\tilde{\phi}, \phi^*(F)]$ opt for a delayed entry in the market respectively as exporters or foreign direct investors. On the contrary, firms with intermediate levels of productivity between $\phi^*(E)$ and $\tilde{\phi}$ and with high levels of productivity, above $\phi^*(F)$, enter immediately the market, choosing exporting and FDI respectively. The dichotomous entry decision of a firm in a static framework becomes a dynamic selection in a dynamic context with firms exporting into the foreign market at different stages of their life, de facto sorting along the time dimension.

The size of the dominance region in figure 3 depends on α through a. As the latter grows larger, the parable expands outward toward the top of the box, enlarging the space of parameter constellation for which FDI results dominant. While affecting only the future profitability of an investment, a higher rate of productivity growth makes FDI more attractive to the firms that have not entered the market yet and are waiting to execute the investment. These firms would reach the trigger profit level for market entry relatively faster than with a slower rate of productivity growth, or equivalently, will grow larger in the same amount of time. Since future profitability has improved, FDI in the future looks more prominent for the same reason FDI looks more attractive to firms with current higher productivity levels. Conditional on delaying entry, firms rather wait for FDI. Exporting could maintain its appeal only if either the transportation costs or the size of its fixed cost would reduce at the same time. To an expansion of the dominance region of FDI corresponds a lessening of the area where the sorting of firms are productivity dependent as the box is of fixed size. In this sense, the proximity concentration trade off is “biased” towards FDI in this dynamic context.

One merit of this approach is that it opens to the possibility of waves in FDI. Market entry in any given period of time is the combined action of firms that do not delay their investments and of firms whose execution time has come. Since an increase or a drop in the productivity growth leads, respectively, to shortening or prolonging the waiting time, waves in greenfield investment can arise.

3 Uncertainty

The decisions firms make over investments carried on in the future are typically characterized by uncertainty. The sources for uncertainty may be several. In the following we present three stylized types of uncertainty that are widely used in theoretical models and which map economic conditions relevant in foreign market entry scenarios. First, following market entry, the firm may be forced to exit the foreign market. Second, the project the firm has planned to undertake at a future date may become suddenly unavailable. Third, the firm is repetitively confronted in the course of its life with the adoption of new technologies, which may cause temporary disruption costs as well as productivity spurts.

While the first two sorts of uncertainty are random events that may occasionally occur during the firm’s existence, the third type of uncertainty is of a continuous nature and potentially challenges the firm repetitively. Therefore, we shall model, as common in the investment literature, these types of uncertainty differently. We shall assume the random events are described by Poisson processes, while we let the continuous type of uncertainty be modeled by means of a Brownian motion.

At any point in time the decision maker knows whether an event which entails benefits and costs of waiting has yet occurred, but because the future remains uncertain her incentives to wait are affected. As a general insight in the investment literature, whether higher uncertainty fosters more action or inaction depends on the specific structure of the problem.[13] We focus here, not primarily on the implications for the timing and the probability of action, but rather on the effects uncertainty has on the firm's internationalization mode.

3.1 Death shock after market entry

Regardless of the firm's productivity and investment mode, the firm may undergo a crisis after market entry and eventually exit the market. Similarly to Melitz (2003), we model this shock as a pure exogenous event described by a Poisson process. Let λ be the mean arrival rate of the death shock in the foreign market: during the infinitesimal time interval dt, the probability that the firm will exit after entry is λdt, and the probability of exit over a period of length s is $1 - e^{-\lambda s}$. The death uncertainty after market entry is independent of the chosen entry mode i.[14] As of time 0 the net present value of the investment i executed at $T(i)$ is

$$\begin{aligned} V_0(i, \phi, T(i), \lambda) &= \int_{T(i)}^{\infty} \pi(i,0) e^{at} e^{-\lambda(t-T(i))} e^{-rt} dt - I(i) e^{-rT(i)} \\ &= e^{-rT(i)} \left(\frac{\pi(i, T(i))}{r + \lambda - a} - I(i) \right). \end{aligned} \tag{15}$$

Again, we assume $r > a$ for a well defined problem. The first product in this integral represents the firm's expected periodical cash-flows for a market entry in $T(i) \geq 0$ and a survival duration of $t - T(i)$ after entry. Indeed, $\pi(i, T(i))$ has to be multiplied by $e^{-\lambda(t-T(i))}$ the probability of surviving up to $t - T(i)$ periods.

Repeating the same steps as above, the level of cash-flow that triggers market entry in each mode is

$$\pi(i, T^*(i, \phi, \lambda)) \geq \left(\frac{r}{r-a} \right) (r - a + \lambda) I(i) \quad (\text{with } = \text{ if } T(i, \phi, \lambda)^* > 0) \tag{16}$$

[13] For a detailed discussion consult Bertola (2010).

[14] We do not consider a case in which the firm can re-enter the foreign market after its death. Hence, the firm holds only one export or FDI option to enter the market. If the option to serve the market was available after the realization of a death shock (in perpetuity), optimal behavior has to be modeled differently. However, as these considerations are more interesting for transitional effects, which are not the focus of this paper, we leave those cases out of consideration.

which is larger than the previous cutoffs triggering entry in absence of uncertainty ($\lambda = 0$). For each mode the new cutoff productivity level results as

$$\phi^*(i,\lambda) = \sqrt[\kappa]{\left(\frac{r-a+\lambda}{r-a}\right)\frac{rI(i)}{M(i)}}. \tag{17}$$

Because the firm anticipates that the time horizon to recoup the fixed cost of the investment (i) may be shorter if there exists the risk of being hit by a shock, immediate entry can only be optimal for a larger payoff at entry. Therefore, the threshold level of initial productivity $\phi^*(i,\lambda)$ commanding no waiting, rises above $\phi^*(i)$. And for a given growth rate of productivity, it also implies more waiting, $T^*(i,\phi,\lambda) > T^*(i,\phi)$. The optimal solution to the foreign market entry problem leading to the highest net value from the investment i becomes

$$O(i,\phi,a,I(i),r,\lambda) = \begin{cases} \left(\frac{r-a}{r-a+\lambda}\right)^{\frac{r}{a}} F(i,\phi) & \phi < \phi^*(i,\lambda) \\ V_0(i,\phi,0,\lambda) & \phi \geq \phi^*(i,\lambda). \end{cases} \tag{18}$$

While the existence of a market specific future death shock ($\lambda > 0$) clearly influences the optimal entry time $T^*(i)$ and the cutoff productivity levels $\phi^*(i,\lambda)$ of each entry mode i, it has no impact on the FDI dominance region that has been derived and depicted in figure 3. This result is easily deduced from the latter maximum value functions and equation (17). Accordingly, within the proximity-concentration cost assumptions condition (12) to (14) are not affected by the availability of a death shock uncertainty, post entry.

The key to understanding this result is to realize that the expected life time of the firm decreases with a positive λ, and therefore periodical cash-flows have to cover the flow equivalent investment costs in a shorter period. In the underlying case, the flow equivalent cost increases to $(r-a+\lambda)I(i)$. Furthermore, in an uncertain world the passage of time can produce valuable information, but in this specific situation waiting longer does not reduce uncertainty. Therefore, the firm does not have any incentive to shorten inaction. If anything, waiting in each mode increases. Since the advantage of the FDI mode over exporting originates in the waiting behavior of firms, the future appeal of FDI does not dissipate. However, because the death shock is independent of the investment mode, the relative waiting time of one mode to the other is unchanged and uncertainty has no effects on the FDI dominance region.

3.2 Death of foreign sales opportunity during waiting period

We model the possibility that a business project may become unavailable while the firm waits for its execution by means of a Poisson process, too. Letting δ be the mean arrival rate of this shock, the probability that the business project is still viable at time s is therefore $\exp(-\delta s)$. The firm bears the whole uncertainty while waiting, but conditional on entry, the future looks exactly like in the situation with no uncertainty. Therefore, the expected net value of the investment for entry at $T(i)$, $V_0(i, \phi, T(i))$, has to be weighted by the probability that the business project is still available at the execution time $T(i)$. Formally

$$\begin{aligned} V_0(i, \phi, T(i), \delta) &= e^{-\delta T(i)} \left[\int_{T(i)}^{\infty} \pi(i, T(i)) e^{-rt} dt - I(i) e^{-rT(i)} \right] \\ &= e^{-(r+\delta)T(i)} \left(\frac{\pi(i, T(i))}{r - a} - I(i) \right) \end{aligned} \tag{19}$$

which, when maximized with respect to the entry time $T(i)$, implies the market entry rule

$$\pi(i, T^*(i, \phi, \delta)) \geq \frac{(r + \delta)(r - a)}{r + \delta - a} I(i), \quad (\text{with } = \text{ if } T(i, \phi, \delta)^* > 0). \tag{20}$$

Again, as a consequence of the Jorgensonian rule, the payoff required for market entry is larger than the one in (7) commanding entry when the firm holds only a "now or never option" in absence of uncertainty. However, it is smaller than the one in (9) triggering market entry when the firm has the option to wait and operates in an environment free of uncertainty. The respective productivity cutoffs which trigger market entry in this case are:

$$\phi^*(i, \phi, \delta) = \sqrt[\kappa]{\frac{(r + \delta)(r - a)}{r + \delta - a} \frac{I(i)}{M(i)}}. \tag{21}$$

Market entry is motivated, in this context, by the likelihood of disappearance of the investment opportunity: reducing the waiting time and insuring oneself a lower, albeit positive, net value of the investment at entry is a superior strategy to regretting a missed investment opportunity for waiting too long. Because the initial productivity threshold commanding market entry decreases, $\phi^*(i, \phi, \delta) < \phi^*(i, \phi)$, the firms within this productivity range are those that are adjusting their entry strategy and hedge against the shock by taking a faster action (immediate entry) than they would in absence of uncertainty. Firms below the threshold $\phi^*(i,\phi,\delta)$ find it nevertheless optimal to wait, but they wait relatively less, $T^*(i,\phi,\delta)$ periods instead of $T^*(i,\phi)$. Firms above this threshold not only do not bear any uncertainty as they are opting for immediate market entry regardless of uncertainty, but they also realize the same net value from the investment i as in the deterministic growth scenario. For any level of productivity, the highest net value of the two investment modes are

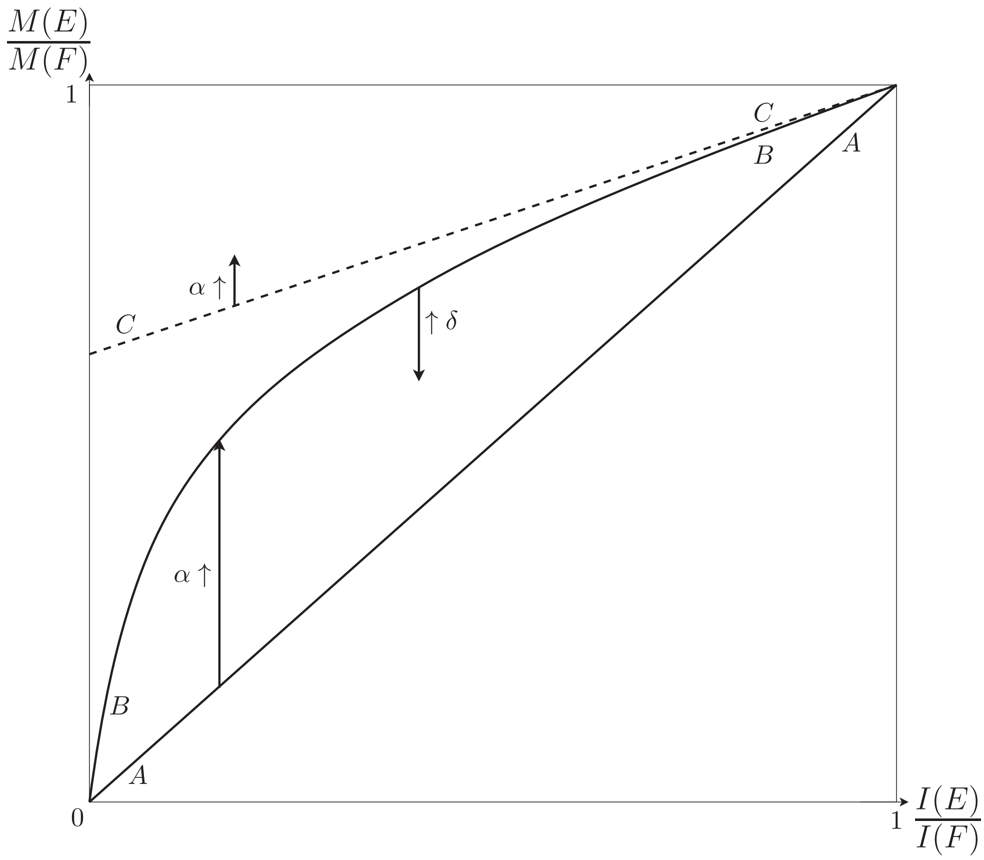
$$O(i,\phi,a,I(i),r,\delta) = \begin{cases} B(i,\delta)\phi^{\frac{r+\delta}{\alpha}} \equiv G(i,\phi,\delta) & \phi < \phi^*(i,\delta) \\ \frac{\pi(i,0)}{r-a} - I(i) \equiv V_0(i,\phi,0) & \phi \geq \phi^*(i,\delta) \end{cases} \tag{22}$$

where $B(i,\delta) = \left(\frac{r+\delta-a}{(r+\delta)(r-a)}\frac{M(i)}{I(i)}\right)^{(r+\delta)/a} \frac{a}{r+\delta-a} I(i)$. $B(i,\delta)$ and $G(i,\phi,\delta)$ converge respectively to $A(i)$ and $F(i,\phi)$ only as δ approaches zero. For $\phi \geq \phi^*(i,\delta)$ the maximum value function solution $V_0(i,\phi,0)$ is exactly as in (11). Therefore, condition (14) for FDI dominance is again unchanged, but condition (13) is changing to

$$G(F,\phi^*(E,\delta),\delta) > G(E,\phi^*(E,\delta),\delta) \qquad \Leftrightarrow \qquad \frac{M(E)}{M(F)} < \frac{I(E)}{I(F)}\left(\frac{I(E)}{I(F)}\right)^{-\frac{a}{r+\delta}} \tag{23}$$

implying that the curve BB shifts inward toward the diagonal AA as δ increases. From equations (21) it is easy to deduce that the AA line representing condition (12) does not change, since the two cutoffs $\phi^*(i,\phi,\delta)$ decrease equally proportional. We visualize the effect of an increase in δ on the derived FDI dominance area in figure 4. Greater uncertainty of this type downsizes the FDI dominance area and counteracts the effects of a larger α. Unlike in the event of a death shock, in this case entry by the investor solves all uncertainty. Therefore the passage of time influences the costs and the benefits of waiting not only because of discounting and growth, but also because it produces relevant information about future uncertainty. Even if the Poisson process is memoryless and the likelihood of the unfavorable event is the same in the course of time, the future, from the perspective of the firm, looks different if the investment is undertaken because acting isolates the firm from the shock. Because firms would wait less, in the sense clarified above, the appeal of FDI is weaker relative to a situation with no uncertainty.

Figure 4: Uncertain foreign sales opportunity during the waiting period



Note: For any relative cost constellation below the BB curve a firm will serve the new foreign market through FDI, independently of the observed productivity level. This FDI dominance region decreases with an increase in δ, the mean arrival rate of a shock that eliminates the availability of a profitable deal on the foreign market, while a firm waits for entry.

3.3 Continuous uncertainty in productivity growth

In our last scenario the firm is confronted with a continuously changing performance which in our case originates from an uncertain productivity evolution. The source for such a continuous uncertainty can be manifold: for example the adoption of new technologies or the implementation of new business practices may cause a temporary boost as well as drops in firm performance.

We model this type of repetitive specific shocks for the firm as a stochastic process based on the following Geometric Brownian motion:

$$d\phi_t = \alpha\phi_t dt + \sigma\phi_t dz_t, \tag{24}$$

where the drift parameter $\alpha \geq 0$ still represents the firm's persistent productivity growth. The diffusion parameter $\sigma \geq 0$ measures the extent of continuous productivity shocks. α and σ are time invariant.[15] dz_t is the increment of a standard Wiener process and uncorrelated across time, with $dz_t = \epsilon_t \sqrt{dt}$ and $\epsilon_t \sim N(0,1)$. Productivity evolves over time independently of whether the firm enters the new market through exports or FDI.

Under these conditions the firm's entry decision becomes a stochastic inter-temporal optimization problem which we solve by using a contingent claims analysis. Stochastic changes in (24) are spanned by an existing asset on a perfect capital market. Asset spanning ensures that when the firm postpones immediate entry, it can invest into a financial asset which is perfectly correlated with the Geometric Brownian motion. This asset is assumed to pay no dividend and, therefore, its complete return can be attributed to its capital gain.[16] With these assumptions the opportunity cost of the firm is equal to the return μ of the spanned asset and comprise an appropriate risk premium. Hence, the new risk-adjusted opportunity rate μ is larger than the previous discount rate r as it includes a risk-premium.[17]

To derive the net present value and the option functions for each entry mode i we have to determine the growth rate of the firm's cash-flow and the appropriate discount factor. Periodical profits $M(i)\phi^\kappa$ are convex in ϕ for $\kappa > 1$. Applying Ito's lemma the expected growth rate a_e is:

$$\mathbb{E}\left(\frac{d\phi^\kappa}{\phi^\kappa}\right)\frac{1}{dt} \equiv a_e = a + \frac{1}{2}\kappa(\kappa-1)\sigma^2, \tag{25}$$

where the second summand accounts for the convexity driven additional effect that leads to a higher expected growth rate ($a_e > a$ for $\kappa > 1$). Because of the convex cash-flow structure the spanned asset likewise has to incorporate convex return streams. In appendix A we derive

[15] This is a needed simplification over the empirical evidence presented in Comin and Mulani (2006) to keep the framework tractable. Nevertheless, we shall perform a comparative static analysis on σ below in our analysis.

[16] The expected return of this spanned asset is $\mu = r + R(\sigma)$ where $R(\sigma)$ is the positive risk premium and $R(0) = 0$, so that $\mu = r$ for a risk free environment. Hence, the distortion parameter σ in equation (24) equals the volatility σ of the spanned asset.

[17] For a detailed discussion refer to Dixit and Pindyck (1994).

the corresponding expected risk-adjusted discount rate μ_e as:

$$\mu_e = r + \kappa(\mu - r). \tag{26}$$

Note that for $\kappa = 1$ (no convexity), $\mu_e = \mu$. For a well defined problem with the value of the project to be bounded, we must have $\mu > a$. The expected net present value of investment i at $t = T(i)$ as of time 0 results as

$$\begin{aligned} V_0^e(i, \phi, T(i), a_e, \mu_e, \sigma) &= \int_{T(i)}^{\infty} \pi(i, 0) e^{a_e t} e^{-\mu_e t} dt - I(i) e^{-\mu_e T(i)} \\ &= \left(\frac{\pi(\phi, T(i))}{r - \kappa(r - (\mu - \alpha)) - \frac{1}{2}\kappa(\kappa - 1)\sigma^2} - I(i) \right) e^{-\mu_e T(i)} \end{aligned} \tag{27}$$

which is identical to equation (6) for $a_e = a$ and $\mu = r$. Again, if the firm can only choose at time $t = 0$ between a "*now or never*" option or entering the market at $T(i)$ it would act according to the *Marshallian rule*:

$$\begin{aligned} & V_0^e(i, \phi, T(i)) \geq 0 \\ \Rightarrow\ & \pi(i, (T(i, \sigma)) \geq (r - \kappa(r - (\mu - \alpha)) - \frac{1}{2}\kappa(\kappa - 1)\sigma^2) I(i), \end{aligned} \tag{28}$$

choosing the mode with the highest expected net present value. This entry condition differs from the entry rule in (7) in two ways. First, for linear periodical cash-flows ($\kappa = 1$) the discount rate for both investment modes i becomes $\mu - \alpha$ which is larger than $r - \alpha$ in (7). This means, in a risky environment characterized by uncertain productivity growth the firm is confronted with a higher user cost of capital due to a higher return for an equivalent investment on the capital market. Hence, in this context, the firm will execute a "*now or never*" investment opportunity i at higher productivity levels because of the additional risk premium.

A second new effect arises from the convexity of profit streams in productivity ($\kappa > 1$). For $\sigma > 0$ the variance of the distribution of productivity increases the further the firm looks into the future.[18] Due to Jensen's inequality, the expected value of the convex profit streams

[18] Given the current state of firm productivity ϕ, the variance of the productivity distribution $\phi(t)$ at t is $\mathbb{VAR}(\phi(t)) = \phi^2 e^{2\gamma t}(e^{\sigma 2t} - 1)$, which is increasing with time.

increases. Indeed, the term $-\frac{1}{2}\kappa(\kappa-1)\sigma^2$ in (27) accounts exactly for this effect, implying the expected net present value of investment i grows larger for larger σ.

In order to neatly assess whether the firm should serve a new market via exports or FDI in an uncertain environment where postponement of entry is possible, it is again necessary to account for the respective option values $F(i,\phi)$. Since productivity evolves stochastically it is no longer possible to derive a unique optimal adoption time as in the deterministic case, because $T^*(i)$ is itself stochastic. It is only possible to derive the expected entry time $\mathbb{E}(T^*(i))$ but this is not expedient for the solution of the firm's decision problem.[19] One possibility to derive the firm's optimal entry choice relies on the contingent claims method commonly used in the related finance literature (cf. Bertola, 2010; Stockey, 2009; Dixit and Pindyck, 1994). Instead of determining the optimal entry time, we derive the optimal entry cutoffs $\phi^*_\sigma(i)$ which suffices to characterize the firm's optimal strategy. Given the multiplicative structure of equation (27), this method proposes a general guess solution for the option values $F(i,\phi)$:[20]

$$F(i,\phi) = A_1(i)\phi^{\beta_1} + A_2(i)\phi^{\beta_2}. \tag{29}$$

[19] Based on the Girsanov theorem the expected entry time in mode i results as

$$\mathbb{E}(T(i,\phi^*_\sigma(i),\phi)) = \begin{cases} \frac{1}{\alpha-\frac{1}{2}\sigma^2}\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right) & \text{if} \quad \alpha > \frac{1}{2}\sigma^2 \\ \infty & \text{if} \quad \alpha \leq \frac{1}{2}\sigma^2, \end{cases}$$

cf. with Karatzas and Shreve (1991, p.196).

[20] Substantially, this method applies Ito's lemma and constructs a risk free portfolio that results in the following differential equation:

$\frac{1}{2}\sigma^2\phi^2\frac{\partial^2 F(i,\phi)}{\partial\phi^2} + (r-(\mu-\alpha))\phi\frac{\partial F(i,\phi)}{\partial\phi} - rF(i,\phi) = 0$

where $F(i,\phi)$ represents the option value function of each entry mode i. The solution of $F(i,\phi)$ in this homogeneous linear equation of second order is a linear combination of two linear independent functions such as in equation (29) where β_1 and β_2 are the roots of the respective fundamental quadratic equation[21]

$$\beta_1 = \frac{1}{2} - \frac{r-(\mu-\alpha)}{\sigma^2} + \sqrt{\left[\frac{r-(\mu-\alpha)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \tag{30}$$

$$\beta_2 = \frac{1}{2} - \frac{r-(\mu-\alpha)}{\sigma^2} - \sqrt{\left[\frac{r-(\mu-\alpha)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0. \tag{31}$$

$F(i,\phi)$ is valid over the productivity range for which it is optimal for the firm to keep the option to invest.

Thus, besides the unknown cutoffs $\phi^*_\sigma(i)$ two further parameter A_1 and A_2 need to be determined. These unknowns are easily identified by introducing three boundary conditions. First, at $\phi^*_\sigma(i)$ the firm is indifferent between holding the option $F(i,\phi^*_\sigma)$ and executing the real investment i with the expected net present present value $V^e_0(i,\phi^*_\sigma)$. This defines the *value-matching* condition:

$$F(i,\phi^*_\sigma) = V^e_0(i,\phi^*_\sigma)). \tag{32}$$

Second, marginal returns of both maximum value functions must be equal in $\phi^*_\sigma(i)$. This is the *smooth pasting* condition:

$$\frac{\partial F(i,\phi^*_\sigma)}{\partial \phi} = \frac{\partial V^e_0(i,\phi^*_\sigma))}{\partial \phi}. \tag{33}$$

Finally, the *limiting* condition

$$F(i,0) = 0, \tag{34}$$

states, that the option is worthless for productivity levels at zero. Due to $\beta_2 < 0$ this last conditions requires $A_2 = 0$ and the option function becomes $F(i,\phi) = A_1(i)\phi^{\beta_1}$.[22] The firm's highest net value of investment for any given level and uncertain growth rate of productivity

[21] Appendix B derives the fundamental quadratic equation as $\Psi = \frac{1}{2}\sigma^2\beta(\beta-1) + (r-(\mu-\alpha))\beta - r = 0$.

[22] Setting $A_2 = 0$ rules out bubble solutions.

is then

$$O(i,\phi,I(i),a_e,\mu_e,\sigma)=\begin{cases}A_1(i)\phi^{\beta_1}\equiv F(i,\phi) & \phi<\phi^*_\sigma(i)\\ V^e_0(i,\phi,0) & \phi\geq\phi^*_\sigma(i)\ ,\end{cases} \tag{35}$$

where $A_1(i)$ is derived based on the boundary conditions.[23] Qualitatively, these functions are equal to the maximum value functions in (11) except that they differ in their quantitative shape. Based on equations (32) to (34) the optimal entry rule for a firm that holds the option to wait turns out to be:

$$V^e_0(i,\phi^*_\sigma,0)\geq\frac{\beta_1}{\beta_1-\kappa}I(i) \tag{36}$$

or

$$\pi(i,\phi^*_\sigma)\geq\frac{\beta_1}{\beta_1-\kappa}I(i)(\mu_e-a_e)$$

where $\frac{\beta_1}{\beta_1-\kappa}$ is referred to as the option multiple. Most importantly, since $\beta_1>1$, immediate entry by the firm is suboptimal as $\frac{\beta_1}{\beta_1-\kappa}>1$. Given uncertain productivity growth, the firm will require a higher project value than the irreversible fixed cost in order to enter the market. Implicitly, the firm requires higher productivity levels in order to execute the investment. This results holds in both entry modes.

How does this option multiple differ in its magnitude compared to the deterministic case? In appendix B we show that for $\sigma=0$, $\beta=\frac{r}{\alpha}$ and the option multiple becomes $\frac{r}{r-a}$. We also prove that the option multiple monotonically increases in σ. Therefore, for $\alpha>0$ and $\sigma>0$ we have $\beta_1<\beta$ and

$$\frac{\beta_1}{\beta_1-\kappa}>\frac{\beta}{\beta-\kappa}. \tag{37}$$

[23] $A_1(i,\sigma)=M(i)^{\frac{\beta_1}{\kappa}}I(i)^{1-\frac{\beta_1}{\kappa}}\left(\frac{1}{(\mu_e-\alpha_e)}\left(\frac{\beta_1(\mu_e-\alpha_e)}{\beta_1-\kappa}\right)^{1-\frac{\beta_1}{\kappa}}-(\mu_e-\alpha_e)\left(\frac{\beta_1(\mu_e-\alpha_e)}{\beta_1-\kappa}\right)^{-\frac{\beta_1}{\kappa}}\right)$

From equation (36), the market entry triggering productivity level in each mode (i) is

$$\phi_\sigma^*(i) = \sqrt[\kappa]{\frac{\beta_1}{\beta_1 - \kappa} \frac{I(i)(\mu_e - \alpha_e)}{M(i)}} \tag{38}$$

and the firm will enter the market at $t = 0$ only if its productivity is higher than $\phi_\sigma^*(i)$. The larger the productivity volatility is the higher becomes the respective entry cutoff $\phi_\sigma^*(i)$ with $\phi_\sigma^*(i) > \phi^*(i)$. Compared to the deterministic scenario uncertainty increases the value of the firm's investment option for productivity levels below $\phi_\sigma^*(i)$, and therefore, the amount of actual investment for those firms decreases. Firms with a productivity level between $\phi^*(i)$ and $\phi_\sigma^*(i)$ postpone action by the expected entry time $\mathbb{E}(T(i, \phi_\sigma^*(i), \phi))$ while firms with a productivity level below $\phi^*(i)$ prolong their waiting time.[24] To sum up, an increase in σ, like an increase in α increases the option function $F(i, \phi)$. However, while expected waiting time before entry decreases in α, an increase in σ entails postponement.

This general result in the investment literature has a straight forward economic intuition. While productivity growth fosters earlier entry, uncertainty incentivizes a firm to postpone market entry and to give up a certain amount of periodical returns for the sake of additional information on how productivity evolves in the next period, even when it would be profitable to enter the market immediately. Uncertainty about the future is the source of the arising value of waiting.

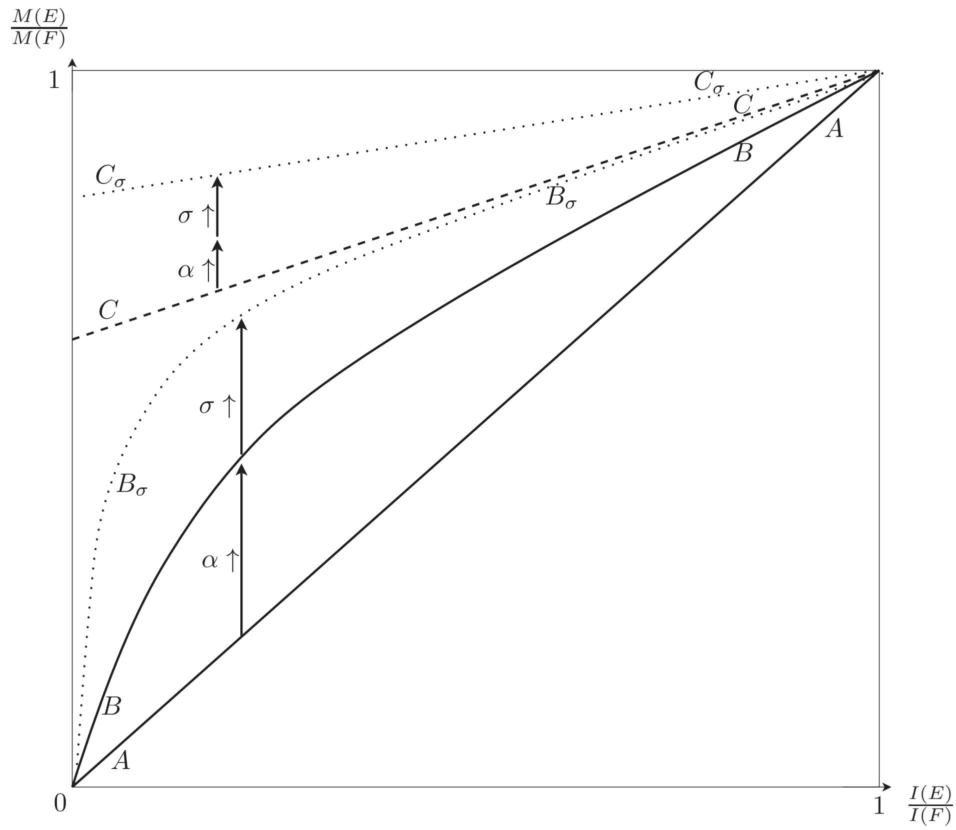
We can determine how continuous uncertainty effects the three conditions (12) to (14) depicted in figure 5. It is interesting to note that the order relation of the two initial entry cutoffs again remains unaffected by the existence of uncertainty. This is because uncertainty influences both productivity cutoffs equally proportional, so that $\phi_\sigma^*(E) < \phi_\sigma^*(F)$ holds for all relative cost constellations that again fulfill condition (12), and the AA curve in figure 5 does not change.[25] However, condition (13) that defines the upper range of the FDI dominance region and condition (14) are affected by uncertainty:

$$F(F, \phi_\sigma^*(E)) > F(E, \phi_\sigma^*(E)) \qquad \Leftrightarrow \qquad \frac{M(E)}{M(F)} < \frac{I(E)}{I(F)} \left[\frac{I(E)}{I(F)}\right]^{-\frac{\kappa}{\beta_1}} \tag{39}$$

[24] In appendix C and D we proof that the expected entry time monotonically increases in σ.

[25] This the consequence of our assumptions that uncertainty is the same regardless of the chosen entry mode.

Figure 5: Continuous Growth Uncertainty and the Choice between Export and FDI



Note: For any relative cost constellation below the $B_\sigma B_\sigma$ curve a firm will serve the new foreign market through FDI, independently of the observed productivity level. This FDI dominance region increases with a rise in σ, the diffusion parameter which determines at any time the extent of deviations from the growth path.

and

$$V_0^e(F, \phi_\sigma^*(F)) > V_0^e(E, \phi_\sigma^*(F)) \quad \Leftrightarrow \quad \frac{M(E)}{M(F)} < \frac{\kappa}{\beta_1} + \frac{\beta_1 - \kappa}{\beta_1}\frac{I(E)}{I(F)} \tag{40}$$

with $\beta_1 < \beta$ for $\sigma > 0$ and $\beta_1 = \beta$ for $\sigma = 0$ and therefore, $C_\sigma C_\sigma$ is above CC.

We visualize the impact of uncertainty on condition (39) and (40) in figure 5. Greater uncertainty shifts the respective locus of points from BB to $B_\sigma B_\sigma$ and from CC to $C_\sigma C_\sigma$. Hence, as the extent of continuous uncertainty increases, the space of relative parameter constellations for which FDI is the optimal entry mode grows larger. Therefore, this last type of uncertainty simply compounds the FDI favoring effect of productivity growth. The economic intuition for the increasing dominance of FDI as the optimal entry mode if uncertainty rises, hinges on higher sunk costs. As a standard real option result, uncertainty raises the option value of both investment alternatives but the increase in the FDI option is strictly larger than the rise in the export option value. [26] Therefore, while the firm postpones entry, marginal returns from the opportunity investment on the capital market must be higher in the FDI mode due to $I(F) > I(E)$. As a consequence, the later a firm executes market entry, the smaller the discounted fixed costs become in the FDI mode relative to exports. Within the proximity-concentration trade-off framework this increases the prospective profitability of FDI for those firms with $\phi < \phi^*_\sigma(i)$.

Overall, an increase in both productivity growth and uncertainty amplifies the dominance of the FDI strategy, but their effect on the expected entry time $\mathbb{E}(T(i, \phi^*_\sigma(i), \phi))$ differs. More precisely, at a given point in time a boost in productivity growth accompanied by a constant extent of volatility (or modest increase) leads to an anticipation of foreign market entry through FDI. In contrast, an increase in volatility along a constant growth rate in productivity (or modest increase) leads to a postponement of the foreign market entry through FDI. Therefore, market entry that is observed in a specific period is shaped by the dynamic cumulative effect of these two forces. For instance, it comprises those firms that are anticipating entry because of a higher productivity growth as well as those firms that, because of an increased uncertainty, had initially retarded entry and for which entry timing has become mature.

According to empirical data, average firm productivity has experienced a strong boost in particular during the second half of the 1980s due to an accelerating IT revolution, accompanied by a modest volatility (Jorgenson, 2001). This temporary positive productivity shock came along with a first surge of FDI flows which lasted until the early 1990s (UNCTAD, 2011). Furthermore, Comin and Philippon (2005) provide evidence for a strong volatility increase on the firm level in the first half of the 1990s, escorted by a drop in FDI flows. Only a subsequent second positive productivity boost at the end of the 1990s and early 2000 (Jorgenson, 2001) gave rise to a second surge of FDI. It is worth to record here that these waves of FDI flows during the last decades emerged along strong changes in productivity growth and volatility. Accounting for the effects of an interplay between α and σ alterations in our model, we can contribute to the explanation of these cyclical FDI flows in the long run and their surge in the short term.

[26] This holds true for productivity levels lower than the cut-off points.

4 Conclusion

We revise the *proximity-concentration* trade-off argument in a dynamic setting and analyze the choice of a firm between export and FDI. Based on the real option approach several different continuous-time scenarios are analyzed. To keep our framework analytically tractable, we build the dynamics of our model on a partial equilibrium version of the monopolistic competitive model. In spite of the necessary simplifications, the main benefit of this approach is, differently to what is commonly assumed in the FDI literature, that firm's entry into the market is postponable. Therefore, both the timing and the mode of entry are determined. We find that the proximity-concentration trade-off is biased towards FDI if entry is associated with sunk costs. Compared to a static optimization problem (such as Helpman et al., 2004) the resulting sorting patterns in the dynamic case turn out to be more rich.

We extend our analysis by accounting for different types of uncertainty, as a firm's decision over investment that may be carried on in the future is typically characterized by uncertainty. Based on Poisson processes we first show that the uncertainty of being hit by a death shock once entered into the market has an impact on the timing of action but does not affect the choice between FDI and exporting. In contrast, if a firm can be hit by a shock that dissipates sales opportunities on a new foreign market while waiting for entry, the appeal of FDI decreases for an increasing share of cost constellations within the proximity-concentration framework. Finally, we model the evolution of firm productivity as a Geometric Brownian motion. Such an independent continuous variation around an expected growth trend is a realistic pattern. The source for such a continuous uncertainty can be manifold, such as the adoption of new technologies and management processes, which may cause a temporary boost as well as drops in firm performance. We find that uncertainty amplifies the effects that we derive for the deterministic case and biases the firm's mode choice toward the FDI strategy. Our findings contribute to the growing monopolistic competition literature that analyze the behavior of multinational enterprises. In contrast to static models a stochastic dynamic analysis provides further explanations why during certain periods we observe foreign market entry to the expense of exports.

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Appendix

A Equivalent Risk-Adjusted Return

Given the Geometric Brownian motion in equation (24) from Ito's lemma we have:

$$\mathbb{E}\left(\frac{d\phi^{\kappa}}{\phi^{\kappa}}\right) = \left(\kappa\phi^{\kappa-1}d\phi + \frac{1}{2}\kappa(\kappa-1)\phi^{\kappa-2}\sigma^2\phi^2 dt\right)/\phi^{\kappa}.$$

Substituting for $d\phi$ leads to

$$\mathbb{E}\left(\frac{d\phi^{\kappa}}{\phi^{\kappa}}\right) = \left(\alpha\kappa + \frac{1}{2}\kappa\sigma^2(\kappa-1)\right)dt + \kappa\sigma dz_t. \tag{A.1}$$

From the quadratic equation in (B.1) which is valid for ϕ^{κ}, with $\Psi(\kappa) = 0$ it follows that

$$\frac{1}{2}\kappa\sigma^2(\kappa-1) = r - (r - (\mu - \alpha))\kappa$$

Hence, the equivalent risk-adjusted rate of return for an exponential variable results as:

$$\mu_e = r + \kappa(\mu - r). \tag{A.2}$$

B The Fundamental Quadratic Equation

Substituting the guess solution $F(i, \phi(\sigma)) = A_1\phi^{\beta}$ and its derivatives into the linear differential equation (29), we receive the fundamental quadratic equation

$$\Psi = \frac{1}{2}\sigma^2\beta(\beta-1) + (r - (\mu - \alpha))\beta - r = 0. \tag{B.1}$$

Consider the total differential

$$\frac{\partial\Psi}{\partial\beta}\frac{\partial\beta}{\partial\sigma} + \frac{\partial\Psi}{\partial\sigma} = 0, \tag{B.2}$$

which can be evaluated at $\beta = \beta_1$. The quadratic equation Ψ increases in β_1 with $\partial\Psi/\partial\beta_1 > 0$. The derivative of Ψ with respect to σ results as

$$\frac{\partial\Psi}{\partial\sigma} = \sigma\beta_1(\beta_1 - 1) > 0, \tag{B.3}$$

because of (30) . From B.2 necessarily we have

$$\frac{\partial \beta_1}{\partial \sigma} < 0. \tag{B.4}$$

Furthermore, the discount rate of periodical profits in equation (27) turns out to be the negative expression of Ψ evaluated at κ. Note that

$$\mu_e - \alpha_e = r - (r - (\mu - \alpha))\kappa - \frac{1}{2}\kappa(\kappa - 1)\sigma^2. \tag{B.5}$$

For bounded results, this discount rate needs to be strictly positive and, hence, κ must lie between the two roots, specifically: $\beta_1 > \kappa > 0$. As a consequence

$$\frac{\partial \left(\frac{\beta_1}{\beta_1 - \kappa}\right)}{\partial \sigma} > 0. \tag{B.6}$$

For $\sigma = 0$, we have $\mu = r$ and from equation (B.1) it follows that

$$\beta_1 = \frac{r}{\alpha} = \beta. \tag{B.7}$$

C Expected Entry Time

By using the Girsanov theorem[27] it is possible to derive the probability density function of the waiting time T_i as

$$f(T(i, \phi^*_\sigma(i)), \phi) = \frac{\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right)}{\sqrt{2\pi\sigma^2 T(i, \phi^*_\sigma(i))^3}}\; e^{-\frac{\left(\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right) - (\alpha - \frac{1}{2}\sigma^2)T(i, \phi^*_\sigma(i))\right)^2}{2\sigma^2 T(i, \phi^*_\sigma(i))}} \tag{C.1}$$

with $\quad \phi^*_\sigma(i) > \phi.$

The Laplace transform of $T(i, \phi^*_\sigma(i))$ is then given by (see Ross, 1996; Proposition 8.4.1)

$$\mathbb{E}\left(e^{-\lambda T(i, \phi^*_\sigma(i))^*}\right) = \int_0^\infty e^{-\lambda T(i, \phi^*_\sigma(i))} f(T(i, \phi^*_\sigma(i))) dT(i, \phi^*_\sigma(i)) \tag{C.2}$$

$$= e^{-\left(\sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda} - (\alpha - \frac{1}{2}\sigma^2)\right)\frac{\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right)}{\sigma^2}} \tag{C.3}$$

[27] A detailed derivation is offered by Karatzas and Shreve (1991, p.196) or by Karlin and Taylor (1975, p.363).

and can be used to determine the expected waiting time as

$$\mathbb{E}(T(i,\phi^*_\sigma(i))) = \int_0^\infty T(i,\phi^*_\sigma(i))f(T(i,\phi^*_\sigma(i)))dT(i,\phi^*_\sigma(i)) \tag{C.4}$$

$$= -\lim_{\lambda\to 0}\frac{\partial\mathbb{E}(e^{-\lambda T(i,\phi^*_\sigma(i))})}{\partial\lambda} = \frac{\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right)}{\alpha-\frac{1}{2}\sigma^2}. \tag{C.5}$$

More precisely

$$\mathbb{E}(T_i(\phi^*_\sigma(i),\phi)) = \begin{cases} \frac{1}{\alpha-\frac{1}{2}\sigma^2}\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right) & \text{if} \quad \alpha > \frac{1}{2}\sigma^2 \\ \\ \infty & \text{if} \quad \alpha \leq \frac{1}{2}\sigma^2 \end{cases} \tag{C.6}$$

$$\text{with} \quad \phi^*_\sigma(i) > \phi \quad \text{and} \quad i\in\{E,F\}.$$

D Expected Entry Time and Comparative Statics

Exploiting the monotonicity of V_0^e in $\phi^*_\sigma(i)$ we prove that $\frac{\partial\phi^*_\sigma(i)}{\partial\sigma} > 0$ and $\frac{\partial\phi^*_\sigma(i)}{\partial\alpha} < 0$ by proving that $\frac{\partial V_0^e(i,\phi^*_\sigma)}{\partial\sigma} > 0$ and $\frac{\partial V_0^e(i,\phi^*_\sigma)}{\partial\alpha} < 0$, respectively.

Rearranging (38) we obtain

$$\frac{M(i)\phi^*_\sigma(i)^\kappa}{\mu_e-\alpha_e} = V_0^e(\phi^*_\sigma(i)) = \frac{\beta_1}{\beta_1-\kappa}I(i). \tag{D.1}$$

The derivative of $V_0^e(\phi^*_\sigma(i))$ with respect to σ results as

$$\frac{\partial V_0^e(\phi^*_\sigma(i))}{\partial\sigma} = \frac{\partial\beta_1}{\partial\sigma}I_i\left(\frac{-\kappa}{(\beta_1-\kappa)^2}\right). \tag{D.2}$$

From B.1 we can derive

$$\frac{\partial\beta_1}{\partial\sigma} = -\frac{\beta_1\sigma(\beta_1-1)}{\sigma^2(\beta_1-\frac{1}{2})+r-(\mu-\alpha)}. \tag{D.3}$$

Substituting into D.2 results in

$$\frac{\partial V_0^e(\phi^*_\sigma(i))}{\partial\sigma} = \frac{V_0^e(\phi^*_\sigma(i))\sigma(\beta_1-1)\kappa}{(\sigma^2(\beta_1-\frac{1}{2})+r-(\mu-\alpha))(\beta_1-\kappa)}. \tag{D.4}$$

For $\beta_1 > 1$ and $\kappa \geq 1$ the numerator is always positive. We can prove that the denominator is also always positive. To do so we rewrite (30) as

$$(\beta_1 - \frac{1}{2})\sigma^2 + r - (\mu - \alpha) = \sigma^2 \sqrt{\left(\frac{r - (\mu - \alpha)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 0. \quad \text{(D.5)}$$

The right hand side of this equation is always positive for $\beta_1 > 1$ and hence $\frac{\partial V_0^e(\phi_\sigma^*(i))}{\partial \sigma} > 0$. Furthermore,

$$\frac{\partial V_0^e(\phi_\sigma^*(i))}{\partial \alpha} = \frac{\partial \beta_1}{\partial \alpha} I_i \left(\frac{-\kappa}{(\beta_1 - \kappa)^2}\right). \quad \text{(D.6)}$$

From D.2 we receive

$$\frac{\partial \beta_1}{\partial \alpha} = \frac{-\beta_1}{(\beta_1 - \frac{1}{2})\sigma^2 + r - (\mu - \alpha)} < 0. \quad \text{(D.7)}$$

Hence we have $\frac{\partial V_i^*}{\partial \alpha} > 0$. Since V_i^* behaves as ϕ_i^* we can state

$$\frac{\partial \phi_\sigma^*(i)}{\partial \alpha} < 0 \quad \wedge \quad \frac{\partial \phi_\sigma^*(i)}{\partial \sigma} > 0. \quad \text{(D.8)}$$

With these results we can consider the following partial derivatives of C.6:

$$\frac{\partial \mathbb{E}(T(i, \phi_\sigma^*(i)))}{\partial \sigma} = \frac{\sigma}{\left(\alpha - \frac{1}{2}\sigma^2\right)^2} \ln\left(\frac{\phi_\sigma^*(i)}{\phi}\right) + \frac{1}{(\alpha - \frac{1}{2}\sigma^2)} \frac{1}{\phi_\sigma^*(i)} \frac{\partial \phi_\sigma^*(i)}{\partial \sigma} > 0 \quad \text{(D.9)}$$

$$\frac{\partial \mathbb{E}(T(i, \phi_\sigma^*(i)))}{\partial \alpha} = -\frac{1}{(\alpha - \frac{1}{2}\sigma^2)^2} \ln\left(\frac{\phi_\sigma^*(i)}{\phi}\right) + \frac{1}{(\alpha - \frac{1}{2}\sigma^2)} \frac{1}{\phi_\sigma^*(i)} \frac{\partial \phi_\sigma^*(i)}{\partial \alpha} < 0. \quad \text{(D.10)}$$

In both modes expected entry time increases in σ and decreases in α.