Uniform Reductions are not that Bad

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Abstract

Both Barrett (1991) and Hoel (1991) show that uniform solutions cannot guarantee that the *IR* constraint is satisfied. This drawback of uniform solutions dramatically reduces feasibility of uniform solutions. However, when uniform reductions are property specified, this conclusion is no longer valid. Compared to Barrett (1991), which proposes a uniform absolute reduction, this paper proposes uniform solutions that are defined as equal percentage reduction compared to some pre-agreement reduction level, in accordance with real world specification. For such a specification, uniform solutions that satisfy *IR* will always exist, establishing a new, and more positive, view upon uniform solutions.

JEL Classification: Q28, H4, H77

Keywords: Uniform reduction obligations, individual rationality.

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1. Introduction

This paper corrects one misunderstanding about uniform reductions obligations applied to international environmental problems (*IEP*) that have given these types of solutions a bad reputation. This is done by showing that, when properly specified, uniform solutions always exists that guarantees that the individual rationality (*IR*) constraint is satisfied without the use of side payments for a general class of international environmental problems. This result is in contrast to common wisdom, built on papers of Barrett (1991) and Hoel (1991), where it is shows that uniform solutions merely accidentally satisfies *IR* when side payments are not reliable. This new insight might explain why uniform solutions remain one of the main default approaches to address international environmental problems.

Caused by the lack of supranational authority empowered to enforce unpopular solution upon sovereign countries, the *IR*-constraint is a very important condition. Basically, individual rationality is a prerequisite for solutions to be part of any politically feasible set. On the other hand, there is a common understanding that side payments are not reliable.[1] The combination of these two conditions leads to certain limitations on the set of feasible solutions, but this paper shows that uniform solutions always are part of this set.

We state two main results. The first is that when reductions are specified relative to the non co-operative outcome, there always exists a uniform reduction that satisfies IR without the use of side-payments. The analysis builds mainly on the reaction function approach, as e.g. specified in Hoel (1992). The implications are that no matter the ex ante information structure, or whether or not cost-curves are common knowledge or private information, the uniform solutions are always reliable. Moreover, it is shown that it is always possible to specify what uniform solution is feasible. The second result is that there always exists an optimal uniform solution that satisfies the *IR*-constraint without the

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[1] Mäler (1993, p.27) mentions that ‘it is somewhat surprising that instances where international environmental problems have been solved with the aid of side-payments are very rare’.

use of side payments, which again is in contrast to the results in Barrett (1991) and Hoel (1991).

Once established that a uniform solution always exists, three different uniform solutions are considered, all yielding a Pareto-improvement over the non co-operative solution, without relying on side payments. The first is the optimal uniform solution, which picks that uniform reduction that maximises the overall welfare, subject to the individual rationality constraint. This solution represents the upper limit on welfare under the uniformity constraint. The second is the least common denominator approach, which has been thoroughly analysed in Brandt (2001) and Eyckmans (1998). This specific solution deserves as a lower bound on the welfare, uniform solutions can guarantee. This result basically comes around because no matter the state of information, this solution will always represent a situation where no participants have any incentive to misrepresent their preferences. In between these upper and lower bounds, other dimensions, like fairness measures, or constraints, could also be applied: This will be the third solution that is considered for a two-country situation. Hence, uniform solutions can be tailored at lest somewhat towards optimality, fairness and truth telling, always satisfying individual rationality, of course const rained by the uniformity constraint.

In the light of the above results, it will no longer come as a surprise that uniform solutions have been focal to many negotiations, especially when taking into account their simplicity.[2] However, one should not exaggerate the merits of uniform solutions, since the major drawback of uniform solutions is there the lack of flexibility. If costs of one country increase, then all countries will have to reduce less under all three uniform solutions specified in this paper. This

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[2] According to Greene (1996) simple expressed commitments involving uniform reduction obligations are usually more negotiable. Falkinger, Hackl and Pruckner (1996) claim that a mechanism must be so simple that a layman understands it Eyckmans (1998) argues that a main reason why uniform solutions are politically feasible is that they are easy to implement. Young (1989) identifies the presence of focal points as one of the main conditions that increases the likelihood of success. Paterson (1996) and Grubb (1996) even argue that differentiation in practice would operate to reduce the overall effectiveness of future climate agreements.

yields the obvious consequence that uniform solutions are most effective when countries are relatively alike.[3]

Uniform obligations demand an equal percentage reduction of all participating countries. Among the more essential agreements can be mentioned: The 1979 convention on long-range transboundary air pollution, which was signed by 32 European countries (and the EEC), the USA and Canada. In 1985 a protocol was added to the convention committing the 21 signatories to reduce sulphur emissions by at least 30% by 1993 as compared to the 1980 emission level (the "30% club"). In 1991 the "protocol on the control of emissions of VOC" (volatile organic compounds) concerned a 30 per cent reduction of the 1988 level of VOC by 1999. The 1989 "protocol on the control of nitrogen oxides" which constitutes a basic obligation to freeze the NO_x -emission at the 1987 level before 1995. The 1987 Montreal Protocol calls for 50% reductions in CFC emissions by the signatories by 1999 compared to 1987 (with a 10-year lag for developing countries).[4] Several agreements that entail differentiated obligations also exist, e.g. the 1976 Bonn Convention for the protection of the Rhine Against Pollution by Chlorides, where the Netherlands, Germany, France and Switzerland agreed to share reduction costs in proportions 34:30:30:6. Last, but not least, the Kyoto-protocol stipulates differentiated reduction in 2008-2012 compared to 1990 for most OECD countries. The last two differentiated reduction regimes are mentioned to point to how general the uses of baseline emissions from which these reduction requirements are calculated are the countries' individual emission levels immediately before the agreement becomes effective. Hence, this way of specifying reduction obligations is very common for international environmental treaties.

To show our point in comparison with the result of Barrett (1991), section 2 states the two different approaches, while in section 3, the point of departure is the original functional form presented in Barrett (1991). Section 4 generalizes

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[3] Tietenberg (1985) reports that for 17 different types of pollutants a uniform percentage emission reduction results in 7-95 percent higher costs to reach a specific environmental target compared to the cost minimizing way.

[4] Greene (1996) notes more that 20 different operational solutions that can be labeled uniform, which all have different merits, and hence uniform approaches surely deserve more attention.

the results building on the reaction function approach of Hoel (1992) and the main results are stated and discussed. Next, important comparative static results are derived in section 5, and these results are used in section 6 to derive upper and lower limits on welfare or uniform solutions. A short presentation of how to incorporate fairness is also done. Finally, section 7 concludes the paper.

2. Model

First, the set-up of an international environmental problem will be presented. The set of countries is *I={1,2,,,,N)*. Each country emits e_i, which causes environmental degradation both domestically and abroad. For simplicity, assume a global emission problem. Hence, each country is affected by the total emission level $e = \sum_i e_i$. Let the emission level in case of no environmental concern at all be e_i^o. Hence, $e_i \in [0, e_i^o]$ and define $E = \{e_1,,,e_N\}$, the set of emission levels. Compared to e_i^o, a country might undertake certain reduction effort. Let $q_i = e_i^o - e_i$ be the actual reduction level of country i. Due to the global pollutant assumption, it is the total reduction level $Q = \sum_i q_i$ is relevant for a country's reduced level of environmental degradation.

There are to equivalent ways of specifying the net-benefit function for a country. The one is to focus on reduction levels (as done by Barrett, 1991) or, alternatively, to focus on emission levels. Since Barrett model is specified in terms of reductions, while the main results of this paper are best expressed in terms of emissions level, both approaches are presented and compared below. [5] First the approach fucussing on emissions is presented. The net-benefit of emission is given by:

$$NB_i(e_i, e) = B_i(e_i) - D_i(e).$$

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[5] For a nice textbook treatment, see e.g. Perman et. al. (1996) and for a more theoretical analysis, see Welsch (1993).

$B_i(e_i)$ measures the benefit from emission. Emissions generate utility as an input in the production and consumption of goods. Alternatively, the benefit functions may be interpreted as the opportunity costs of abatement of countries (where lower emissions imply higher opportunity costs). $D_i(e)$ measures the damage from total emission. Standard assumptions of strictly concave *NB* functions comprising strictly concave benefit and strictly convex damage functions are made: $B_i^{'}(e_i) > 0$ and $B_i^{''}(e_i) \leq 0$ while $D_i^{'}(e) > 0, \mathrm{D}_i^{''}(e) \geq 0.$ Let $\mathrm{B} = \{B_1,,, B_N\}$ and $\Omega = \{C_1,,, C_N\}$ be the set of strict concave benefit function and strict convex environmental cost functions, respectively.

Define an emission game $g \in \Gamma = \{I, E, \mathrm{B}, \Omega\}$. In this way, an emission game (or an IEP) γ consists of a profile of players, their NB-functions and the individual emission levels. In what follows, the number of players and the feasible emission levels is fixed at every profile. Hence, the γ's differ with respect to cost and benefit functions and define a general class of IEP with strict concave benefit function and strict convex environmental cost functions, respectively. In this analysis focus will be exclusively on cost-differences, and it is assumed that all countries benefit functions are equal. The analysis is equally valid if focussing on differences on the benefit side.

Specifications in terms of reduction levels are also possible. In this version, $B_i(Q)$ measures the benefit to country i from total reduction, Q. The benefit is derived from reduced damages from controling emission. On the other hand, costs from controlling emission only dependents on own reduction, q_i, and is measured by $C_i(q_i)$. Hence, the net-benefit to country i amounts to:

$$NB_i(q_i, Q) = B_i(Q) - C_i(q_i) .$$

Here too, we make the standard assumptions on the functions:

$B_i^{'}(Q) > 0$ and $B_i^{''}(Q) \leq 0$ while $C_i^{'}(q_i) > 0, \mathrm{C}_i^{''}(q_i) \geq 0.$

The comparison between the two approaches is best done be identifying that they represent the same information. The first identity is:

$$\frac{\partial B_i(Q)}{\partial Q} = \frac{\partial D_i(e)}{\partial e} \text{ for } \sum \mathrm{q}_\mathrm{i} = \sum (e_i^o - e_i).$$

The marginal benefit from increased reduction is equal to the marginal damage from increased reduction, at the point where emission levels corresponds to reduction levels.

The second is:

$$\frac{\partial C_i(q_i)}{\partial q_i} = \frac{\partial B_i(e_i)}{\partial e_i} \text{ for } \mathrm{q}_\mathrm{i} = e_i^o - e_i.$$

The marginal cost from increased reduction is equal to the marginal benefit from increased emission (the gain from avoiding additional abatemetn by a marginal increase in emission). This shows that the two approaches, using reduction levels or using emission levels, of course, are equivivalent and their relative merit dependent on the circumstances.

If each country behaves non co-operatively, it maximises its respective net-benefit function with respect to its own emissions, e_i, considering only damages in its own country but not those abroad, or alternatively, not considering the public good character of its own reduction effort on the other countries' well being.

Formally, the definition of the non co-operative levels of emissions and reductions are:

$e_i^{nc} = \arg\{D_i^{'}(e_i, e_{-i}^{nc}) = B_i^{'}(e_i)\}$ and $q_i^{nc} = \arg\{B_i^{'}(q_i, q_{-i}^{nc}) = C_i^{'}(q_i)\}$.

Where $e_{-i} = \{e_1,,e_{i-1},e_{i+1},,e_N\}$ and $q_{-i} = \{q_1,,q_{i-1},q_{i+1},,q_N\}$.

As done throughout the literature, assume that in case of no coordination, these levels will result.[6]

Hereafter, define uniform reductions as follows:
Absolute uniform reductions (Barrett, 1991):
$q_i = q_j, \forall i, j \in I$.
Relative uniform solutions:
$e_i = \alpha e_i^{nc}, \alpha \in (0,1), \forall i \in I$.

Where 1-α is the equal percentage reduction for all countries. E.g. α=0.7 requires a 30% reduction in a specified future date compared to observed past emissions of any signaturing country.

The first simply states that countries reductions are equalized. This type of uniform reduction requires that each country reduce the same in absolute terms. In Barrett (1991), this can at first be justified since the countries are of equal size. Once the countries differ in any respect, like in Barrett's model where the costs of the two countries differ, it, however, generates strange results as will be clear in section 3. The second approach, on the other hand takes actual emissions into account. In this way it take into account that e.g. cost differences has moved else wise symmetric countries onto different emission-paths. Hence, the main difference between the two approaches is on what basis the reduction is compared. In Barrett, the uniform reduction is compared to e^o, while in the second approach, uniform reduction is compared to the actual emission level (the non co-operative level, e_i^{nc}). As long as these two differs, the two approaches yield different result. This will be clear in section 3, where an example is presented.

A very important uniform reduction is the preferred uniform reduction of a country which also is denoted the uniform peak of country i.
Define the peak of country i: $\alpha_i^P = \arg\max\{B_i(\alpha e_i^{nc}) - D_i(\sum \alpha e_j^{nc})\}$.
Due to the assumptions on B and D, there always exists a unique $\alpha_i^P < 1$ for all countries.[7] It is in the same way possible to define a peak with respect to q.

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[6] This is a common observation; see e.g. Hoel (1998).

Note that in order for a country to being able to calculate its peak it only need knowledge about its own benefit and damage function and the observed emission levels of the other countries.

The basic claim of this paper is that the most fundamental requirement that a solution must satisfy in order have any change on yielding any progress to an *IEP* is that of *IR*. The relevance of this requirement is e.g. noted by Sandler (1997): ‘Successful collective actions require that all participants perceive a net benefit. This simple realization is often forgotten'. In all, this paper expressed the view that *IR* is a necessary condition that cannot be questioned, if sovereign countries are to engage in a voluntarily agreement. It can only be expected that a country enters an agreement that yields as least as much as in the case of no coordinated actions at all. In this way, $NB_i(e^{nc})$ serves as a defintion of what acountry at least should expect from enterning an agreement.

Define individual rationality (*IR*): A vector *e* satisfies *IR* if $NB_i(e) - NB_i(e^{nc}) \geq 0, \forall i \in I$. Alternatively: $NB_i(q) - NB_i(q^{nc}) \geq 0, \forall i \in I$.

In this paper, this ex ante *IR* specification is used. This can be viewed as in two ways: Firstly, if one country leaves the agreement breaks down and we are back to the non co-operative situation. Since this is known in advance, there will be no agreement if one countries *IR* is not satisfied. Secondly, it can be viewed as a minimum requirement (in case that a country leaves the agreement, this is the worst that can happen).

After having established the basic set-up, focus will in the next section be to compare the Barrett (1991) specification with the relative uniform reductions approach, in order to clarify why the first does not guarantees *IR*, while the second approach does.

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[7] For more on this, see Brandt (2001).

3. Comparison of different specifications, an example

First it is instructive to reproduce the specification of Barrett (1991). His original specification consists of the following functional forms:

$$B_i(Q) = b_i[10Q - \frac{Q^2}{4}] \text{ and } C_i(q_i) = \frac{c_i q^2}{2}.$$

Hence, the net-benefit to country i amounts to:

$$NB_i(q_1, Q) = B_i(Q) - C_i(q_i) = b_i[a_i Q - \frac{Q^2}{4}] - \frac{c_i q^2}{2}.$$

Moreover, in Barrett's cases, he assumes that $N=2$ and $a_1=a_2=10$.

Table 1: Barrett's (1991) case with $c_1=1$, $c_2=2$ and $b_1=b_2=1$

	Country 1			**Country 2**		
	Abatement	**Net benefit**	***DF1***[a]	**Abatement**	**Net benefit**	***DF 2***[b]
Non co-operative	5.74	51.09	0.00	2.87	59.16	0.00
1 chooses $q_1=q_2$	6.67	66.67	15.58	6.67	44.42	-14.74
2 chooses $q_1=q_2$	5.00	62.50	11.41	5.00	50.00	-9.16

[a] $DF1 = NB_1(q_1, q_2) - NB_1(q_1^{nc}, q_2^{nc})$. [b] $DF2 = NB_2(q_1, q_2) - NB_2(q_1^{nc}, q_2^{nc})$.

The first problem with Barrett's specification is that it generates strange results. From table 1, it can be concluded that the preferred uniform solution for country 2, the one where both reduce 5, yields a loss to country 2 of 9.16 compared to the non co-operative situation. This obviously implies that no uniform solutions satisfy the individually constraint for country 2, which is the point made by Barrett. However, what lies behind this result and what does this solution imply? While country 2 abates 74.2% *more* than under the non co-operative solution, country 1 abates 12.9% *less* than under the non co-operative solution.[8] These figures reveal that this can hardly be called a uniform reduction once compared to the situation before the agreement is made, which will be the basis to evaluate uniform reductions in this paper.

Another problem with Barrett's specification is that it only focuses on reduction levels, while real-world specifications are, as indicated in the introduction, often percentage reduction compared to actual emission-levels. Since e^o is not specified (or defined), we cannot simply translate reduction obligations into emission levels. This, however, does not establish a real problem for the comparison. This is seen by transforming Barrett's model into a specification in terms of emissions, and more specific, in terms of uniform reductions compared to the non co-operative situation. Let e_i^o be emission level without any environmental concern, e_i the actual emission level and finally, hence $q_i = e_i^o - e_i$, the actual reduction level. In case of no reduction obligations, each country's reduction is $q_i^{nc} = e_i^o - e_i^{nc}$. Arbitrarily, we choose $e_1^o = e_2^o = 20$. Let the reduction obligation be given as $\alpha<1$, resulting in reduction of $q_i = e_i^o - \alpha e_i^{nc}$. The resulting simulations of this specification is shown in table 2.

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[8] This solution is, for obvious reasons never acceptable for country 2, and hence, the uniform solution presented here is not generating any progress.

Table 2: Barrett's example with the relative uniform approach

c_2	a_1^P	q_1	q_2	NB_1	NB_2	q_1^{nc}	q_2^{nc}	$DF1$	$DF2$
1	0.89	6.65	6.65	66.67	66.67	5.00	5.00	4.17	4.17
2	0.86	7.71	5.27	57.95	59.92	5.71	2.86	6.86	0.76
3	0.84	8.24	4.88	54.22	52.44	6.00	2.00	8.22	-5.56
5	0.84	8.45	4.25	50.98	41.52	6.25	1.25	9.57	-15.51
c_2	a_2^P	q_1	q_2	NB_1	NB_2	q_1^{nc}	q_2^{nc}	$DF1$	$DF2$
1	0.89	6.65	6.65	66.67	66.67	5.00	5.00	4.17	4.17
2	0.92	6.85	4.24	56.69	62.19	5.71	2.86	5.60	3.03
3	0.94	6.84	3.08	51.21	60.37	6.00	2.00	5.21	2.37
5	0.96	6.80	2.00	45.52	58.64	6.25	1.25	4.11	1.61

Again let $N=2$ and $a_1=a_2=10$, with $c_1=1$, $b_1=b_2=1$ and c_2 varying from 1 to 5.

Comparing the two tables, the most striking difference is that IR in table 2 now is satisfied for any preferred choice of country 2. Hence, *uniform solutions now exists that satisfied IR.*

The reason for this is that reductions now are compared to the non co-operative actions coupled with the fact that there always exist gains for some reductions above the non co-operative level. Compared to Barrett (1991), individual rationality is satisfied. Of special interests is that, as seen in table 2, only the peak of the highest cost type can guarantee *IR*: In case of high cost-difference the peak of country 1 leads to a loss for country 2.[9] Hence, although a uniform solution exists that satisfy IR, since they only can guarantee *IR*, when the peak of the highest cost-country is used, it might not bring around much reduction.[10] The last result hinges on the fact that a_i^p approaches 1, as the cost parameter, c_i increases sufficiently.

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[9] This is a general result, see Brandt (2001).

[10] Or more correct, uniform reduction around the peak of the highest cost country.

4. Reaction functions

Table 2 reveals that the alternative specification used in this paper yields *IR* for the specific problem analysed by Barrett. It is now time to generalize this result and to show that it is valid for all emission games with concave *NB*-functions. The most convenient to proceed is to use a reaction function specification. Let the countries maximize $NB_i(e_i, e) = B_i(e_i) - D_i(e)$. Define the best reply function, or the reaction function of country i, as the function that relates the optimal choice of country i to the choice of the other countries, $R_i = e_i(e_{-i})$. The slope of the reaction function is found by inserting the reaction function into the first order condition $B_i^{'}(e_i(e_{-i})) = D_i^{'}(e_i(e_{-i}), e_{-i})$. Differentiating this expression with respect to e_{-i}, and rearranging yields: $R_i^{'} = \frac{\partial e_i(e_{-i})}{\partial e_{-i}} = \frac{D_i^{''}}{B_i^{''} - D_i^{''}}$.

Since $D_i^{''} \geq 0$ and $B_i^{''} \leq 0$, we have that $-1 \leq R_i^{'} \leq 0$. Given the assumptions on the cost and damage function, a unique interior solution always exists.[11] Define $e^{nc} = \{e_1^{nc}, \ldots, e_N^{nc}\}$ as the non-cooperative (and unique) equilibrium level of emissions, given by the intersection of the reaction functions.

Define $I_i = \{(e_i, e_{-i}) \mid NB_i(e_i, e_{-i}) = k\}$ as the set of iso-net-benefit lines for country *i*, i.e. I_i is the set of emission levels that yields constant net benefit for country *i* in the $(e_i, e_{-i})-\text{plane}$. Given the assumption on *C* and *B*, the I_i's are concave and continuous and their 'maximum' defines the reaction function.

Lower I_i implies higher NB_i, since it implies points of unchanged emission for country *i*, but lower

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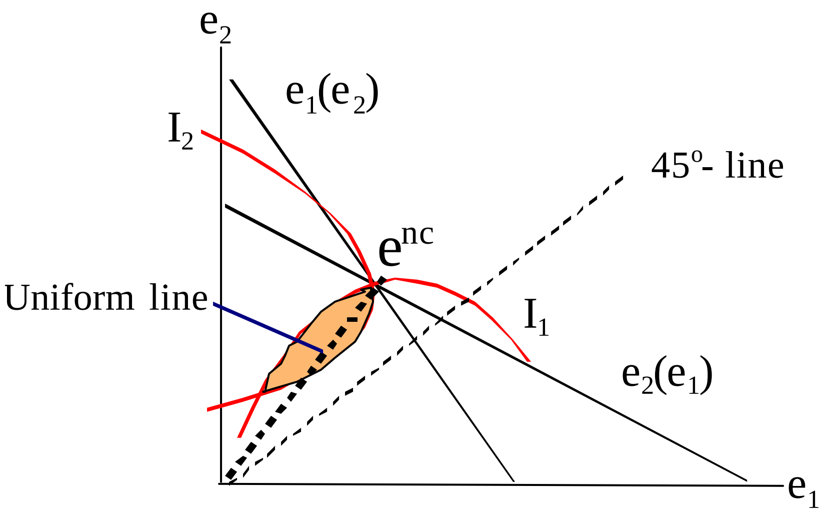
[11] For more on conditions for uniqueness and interior solutions, see Hoel (1991). A thorough discussion of the shape of benefit and damage curves can be found in most books on environmental economics, for a sound discussion, se e.g. Perman et. al (1999). Justifications for Nash-equilibrium under no agreement see e.g. Hoel (1991).

Figure 1: Uniform solutions



emission for its opponents. All this is summerized in figure 1. Most importantly, there always exists, under the assumptions made on *B* and *C*, a set a points that yields a Pareto-improvement over e^{nc}, situated to the 'South West' of e^{nc}, in between the iso net-benefit functions through e^{nc}[12]. In figure 1, the shaded area in figure 1 represents Pareto-improvements compared to e^{nc}. The most important observation in this section is stated in the lemma below:

Lemma 1: The straight line from e^{nc} to the origin always enters the set of Pareto-improvements over e^{nc} for all $g \in \Gamma$.

The proof is straightforward: Recall that at e^{nc}, $\frac{\partial NB_i(e^{nc})}{\partial e_i} = 0$ implying that marginal changes in e_i does not alter the net benefit received for country *i*. On the other hand, $\frac{\partial NB_i(e^{nc})}{\partial e_j} < 0$, since $D_i'(e) > 0$. Hence, for sufficient small (including equal) reductions in emissions, the net benefit is increased for country *i*.

Given the assumptions on *B* and *D*, there always exists a Pareto improvement set as specified in figure 1. The trick here is that at e^{nc}, the iso net benefit lines of country 1 and 2 are horisontal and vertical, respectively, while line from the

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[12] This result follow from standard reaction function analysis like presented in Hoel (1991) or e.g. Tirole (1988).

origin to e^{nc} is positive sloping. This line always enters this set. An example of this is presented in figure 1.

Now it is possible to state the main results concerning the type of uniform solutions that are specified relative to e^{nc}.

Proposition 1: When uniform reductions are specified as $e_{\mathrm{i}} = a e_i^{nc}$, then for all $g \in \Gamma$, there exist an $\alpha \in (0,1)$ such that$[B_i(a e_i^{nc}) - D_i(\sum a e_j^{nc})] - [B_i(e_i^{nc}) - D_i(\sum e_j^{nc})] > 0$ without any use of side payments.

Recall that $e_{\mathrm{i}} = a e_i^{nc}$. Increasing a from 0 to 1, yields a straight line from the origin to e^{nc}. By use of lemma 1, the proof is established.

For any *IEP* with strict concave *NB*, and with emissions target specified as $e_{\mathrm{i}} = a e_i^{nc}$, there always exists a uniform solution that satisfies the individual rationlity constraint without the use of side payments.

Note that the 45^{o}-line in figure 1 is the set of uniform solutions for the ‘Barrett’ specification, when $e_1^o = e_2^o$. In this case, no uniform solution enters the Pareto-improvement set. Given his specification, uniform solutions only accidently satisfy the *IR*-constraint. On the other hand, given the uniform reductions are specified relative to the non co-operative solution, there always exists a whole set of Pareto-improving uniform solutions.

Table 2 and figure 1 both indicate that certain uniform solutions satisfy IR, while others do not. Moreover, it should be clear that the higher the reduction levels, the smaller, ceteris paribus, the possibility that this reduction level satisfies IR. In order for a better description of this connection, we will now analyse how the set of uniform solution that satisfy IR depends on changes in the cost-structure of a country.

5. Comparative static results

Table 2 indicates that the higher the costs of country 2, the higher its peak, implying that it prefers smaller reduction obligations. This result can be generalized using comparative static analysis, which will be will be done by focussing on graphical and intuitive reasoning. In order to focus on changes in costs, define a shift parameter, b_i that shifts the B_i function.

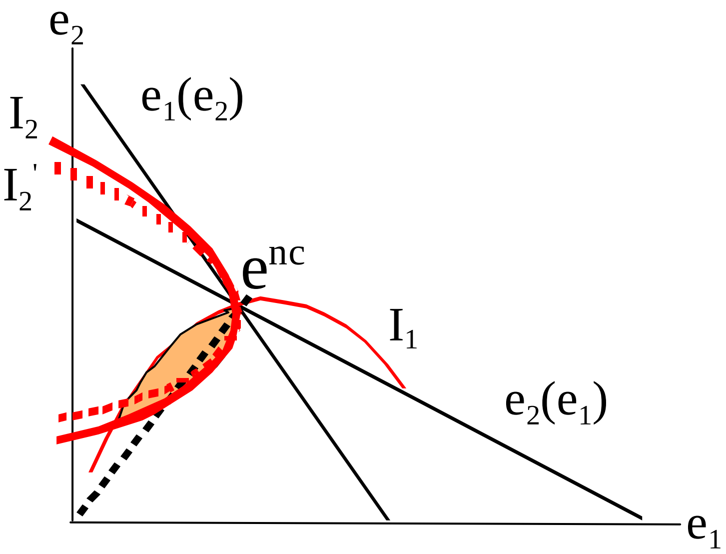
Definition: Let $B_i = B_i(e_i, b_i)$ with b_i a shift parameter. Let higher b_i mean higher abatement costs: $b_i' > b_i \Rightarrow B_i(e_i, b_i') > B_i(e_i, b_i)$ for all e_i.

The idea is that higher b_i imply an upward shift in the abatement cost curve. Since the benefit from emission is the opportunity cost of abatement, higher costs implies higher benefits from emissions.

Result 1: The set of individual rational uniform solutions is non-increasing with increasing b_i.

To see this, note that the higher costs, the more concave the iso net-benefit curves. Recall that the I_i-curves measure the level of compensation necessary in form of changes in e_i when e_{-i} changes in order for country i to be equally well of (a decrease in e_1 needs some decrease in e_2 to make country 1 equally well of). As the costs of a country increase more compensation it needed, which amounts to say that the higher the costs, the more concave the iso net-benefit curves. This is also shown in figure 2, where the new 'more concave' curve being is denoted I_2'. Hence, the area that country 2 prefers shrinks for increasing c_2. Since the Pareto-improvement set is situated between the iso net-benefit functions, the result follows immediate. Notice that for sufficient high costs, there is almost no possibility for further improvements.

Figure 2: Increasing costs



Result 2: $\frac{\partial a_i^p}{\partial b_i} > 0$ implying that $a_i^p \to 1$ for $b_i \to \infty$.

Due to the assumed curvature of *B* and *D*, an upward shift in B_i will always imply a higher peak for that country and for obvious reasons: While the damage side is unchanged, the benefits from indidual emissions in increased and the result follows. This also gives an explanation of point 1 above. From a point, where costs are equal, increasing costs of one, e.g. country 2, the Pareto-improvement area is a smaller and smaller subset of the original set.

Result 3: The higher the cost-difference, the higher the differences in the peaks: $\frac{\partial \left| a_i^p - a_j^p \right|}{\partial b_j} > 0$.

Since the peak of the other countries is not influenced by the cost structure of other countries, this result follows from result 2.

Result 3 implies that the difference between the peaks increases, as the cost differences increase. This is another way of showing that the relative merit of the uniform solutions reduces when the cost differences are high (of equivalently, the benefit differences are high).[13] Combining these points reveals that with increasing cost-differences, the disagreement over what uniform solution to use increases as well. This amounts to say (which is a well-known observation, normally based on the lack of cost-efficiency) that uniform solutions are most easy to implement, when costs are equal.

6. Upper and lower bound on welfare

Proposition 1 state that for all γ, it is possible to find a uniform solution that satisfies *IR*. Note that it does not say that any uniform solutions necessarily do. Hence, we are left with the problem of specifying what uniform to choose. In this section, we will present two types of uniform solution, that deserve as an upper and a lower bound on welfare given focus is on uniform solutions. The upper bound is the uniform solution that maximizes the collective net benefit under the *IR*-constraint. The lower bound is that (unique) uniform solution that, no matter γ, always secures *IR*. In this way, the upper bound is reachable when the exact γ is known in advance, while the lower bound is important in cases when the emission game could be any, hence, in cases of e.g. total ex ante private information about the *B*'s.

6.1 The optimal uniform solution, upper bound on welfare

The upper bound on welfare under the uniformity assumption can be determined by considering the optimal uniform solution, i.e. that uniform solution that maximises overall welfare under the *IR*-constraint. Now define $NB_i(a) = B_i(ae_i^{nc}) - D_i(\sum_j ae_j^{nc})$ as the net benefit to country i when all countries are subject to the same relative reduction of $e_j = ae_j^{nc}$.

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[13] However, the same could be said about differentiated solutions. See section 7.

To find the optimal uniform solution, we have to solve the following maximisation program:

$$\underset{a}{Max}\left(\sum_i (NB_i(\boldsymbol{a})\right)$$
$$S.t\ \boldsymbol{a} \geq 0$$
$$S.t.\ \boldsymbol{a} < 1$$
$$S.t.\ NB_i(e_i^{nc}) - NB_i(\boldsymbol{a}) \leq 0$$

Before formulating the Kuhn-Tucker conditions for this problem, recall that $0 \leq \boldsymbol{a} < 1$.
Hence, the Lagrangian becomes $\ell = \sum_i NB_1(\boldsymbol{a}) - \sum_i l_i(NB_1(\boldsymbol{a}))$, where l_i are the langrage multipliers on the *IR*-constraint. The Kuhn-Tucker conditions reads:

$$\sum_i (1 - l_i)\frac{\partial NB_1(\boldsymbol{a})}{\partial \boldsymbol{a}} = 0 \qquad [a]$$
$$NB_i(\boldsymbol{a}) \geq 0$$
$$l_i\,(NB_i(\boldsymbol{a})) = 0, \forall \mathrm{i} \in \mathrm{I}$$
$$l_i \geq 0, \forall \mathrm{i} \in \mathrm{I}$$

Two cases can be identified, one where the *IR* are all non-binding and one where non-binds.

Case 1: " l =0

Then [*a*] becomes

$$\sum_i \frac{\partial NB_i(\boldsymbol{a})}{\partial \boldsymbol{a}} = 0$$
$$NB_i(\acute{a}) > 0,\ \forall i \in I$$

First, obviously, the solution must lie between the highest and the lowest peak, since all *NB*-functions are strict concave. As a is increased from the lowest peak, all countries with higher peaks will increase their *NB* and all countries with lower peaks will decrease their *NB*. At some point, the increase and decrease will outweigh each other, and this will exactly happen when $\sum_i \frac{\partial NB_1(a)}{\partial a} = 0$.

Case 2: $ l ¹ 0

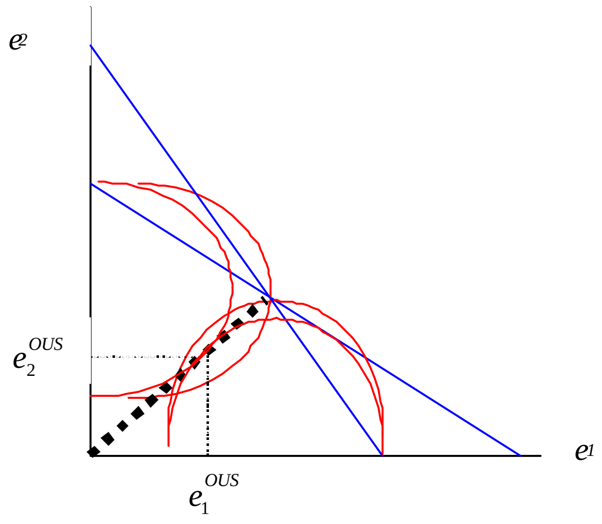
$$\sum_i \frac{\partial NB_1(a)}{\partial a} < 0$$

$$NB_i(a) = 0$$

$$l_i > 0$$

If a NB is binding before an interoiur solution is reached, $\sum \left(\frac{\partial NB_i}{\partial a} \right) < 0$. This means that reducing α would yield an increase in total *NB*, but this option is not feasible since for at least one country *NB* binds, restricting the overall welfare. Comparing with the discussion in section 4, this is most likely to exist when large differences in costs exists, or when one country has much higher costs than the others.[14]

Figure 3: The optimal uniform solution



Combining the above with proposition 1, a very interesting result emerges:

Proposition 2: There always exists an optimal uniform solution that satisfied *IR* without the use of side payments.

Due to proposition 1, we know that there always exists a solution to the optimal uniform solution. The optimal uniform solution is at the point in the plan where the two indifference-

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[14] In the last case, welfare could presumably be increased considerably by not including the high cost country.

curves are tangent to the uniform line. We can identify two cases, one where individual rationality constraint binds, and one where it does not. In the first case, the *OUS* lies on the 'contract curve', i.e. the tangency point of the 45°-line and the two indifference curves, as shown in figure 3, indicated by the point (e_1^{OUS}, e_2^{OUS}). If the *IR*-constraint binds, then points on the 'contract curve' are unlikely obtainable without side payments, but it still yields a Pareto improvement compared to e^{nc}.[15] Here, the optimal uniform solution I identified as the point where the uniform line cuts the iso-welfare line of the country with the binding IR-constraint Note that these arguments are equally valid for a n country solution. This result is again in contrast to Barrett (1991), where constrained optimal solutions might not exists without the use of side payments.

6.2 The least common denominator, the lower bound on welfare

The least common denominator (LCD) solution simply picks the highest peak, i.e. the peak implying the lowest overall reduction.

Formally: let $\boldsymbol{a}^{LCD} = \max[\boldsymbol{a}_1^P, \boldsymbol{a}_2^P,,, \boldsymbol{a}_n^P]$.
The solution that always picks $\boldsymbol{a}^{LCD}$ is called the least common denominator solution.

This solution always satisfies *IR*[16]: Take e.g. the solution where the second largest peak $\boldsymbol{a}_{n-1}^P$ is chosen. We know that $NB_n(\boldsymbol{a}_n^P) - NB_n(\boldsymbol{a}_{n-1}^P) < 0$, while $NB_i(\boldsymbol{a}_n^P) - NB_i(e^{nc}) > 0$, for all i. comparing these two equations it is evident that the only peak that always guarantees IR is the highest peak.

It is shown in Brandt (2001) that the least common denominator solution is the only solution that under private information about reduction costs satisfies the IR constraint for any combination of costs and benefits. This then implies that using this solution, we are certain that the IR also ex-post is satisfied, and in

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[15] By reducing $\boldsymbol{a}$ slightly, both countries can be made strictly better of.

[16] By always, it is meant for any strict convex cost function.

this way, no country will leave the agreement on basis of a violation of the IR-constraint.

6.3 When lower and when upper bound?

If, along the way if implementing the obligation, a country realizes that it receives a net-loss, it will presumably leave the agreement, which makes the agreement unstable. If we stick to the hypothesis that only a solution that ex-ante can guarantee IR has the potential of being a candidate for a stable solution, this can serve as a way to select between different uniform solutions.

The optimal uniform solution represents the upper bound on welfare, when restricted to uniform solutions that without the use of side-payments guarantees *IR*. If there is full information ex-ante, this solution is feasible in the sense that it needs full information to secure that IR is satisfied ex post. That is, if all the *C* and *B* are perfectly known in advance, it will also ex post guarantee *IR*. This might be especially severe when *IR* is binding, i.e., when cost differences are large. Due to result 2 and 3 in section 5, we can conclude that it is more likely that the *IR* binds, when cost-differences are high. Due to result 1, the lower the cost difference, the higher reductions are possible.

The *LCD*–solution on the other hand guarantees that no matter the knowledge, if each country simply announces its preferred peak, this solution always guarantees *IR*. (see Brandt 2001). Hence, it seems most appropriate under private information of reduction costs. Due to result 2, we know that when cost-differences are high (or costs in general are high), then it does not yield much overall reduction.

6.4 Fairness and uniformity

Each countries peak deserves as a natural focal point for a country when entering a negotiation. We saw in the preceding section that when cost-differences exists, the peaks will turn out different, which obviously complicates negotiations and creates the need for a rule that chooses a uniform reduction on basis of certain fairness measures. In our model, a fairness measure should for obvious reasons propose a uniform reduction in between the highest and lowest peak. Furthermore, it must always guarantee *IR*. As an example of such a rule, here only specified for a two-country situation, is the following fairness rule:

Choose $\boldsymbol{a} \in (\boldsymbol{a}_1^p, \boldsymbol{a}_2^p)$ such that: $\dfrac{NB_1(\boldsymbol{a}) - NB_1(e^{nc})}{NB_2(\boldsymbol{a}) - NB_2(e^{nc})} = \dfrac{NB_1(\boldsymbol{a}_1^p) - NB_1(e^{nc})}{NB_2(\boldsymbol{a}_2^p) - NB_2(e^{nc})}$.

This rule is based on the idea that it equalizes what each country could have got at best relative to what it actually gets. This fairness rule has three important properties:

1. If countries are equal, it treats them equally.
2. It always is individual rational.
3. The resulting uniform reductions are always between the lowest and the highest peak.

While the two first points are rather obvious, the third needs some words of explanation. At the highest peak, this country gets all it can possibly get, while the other gets less. By reducing α slightly, the high-cost country get relative less, the low cost country gets relative more until somewhere in-between the peaks described by the above expression. Total welfare is also increased compared to taking the highest peak. Simply because a reduction in α from this peak, the increase in *NB* for the low cost country is higher than the decrease in *NB* for high-cost country (due to the concavity of the net-benefit functions). Hence it establishes a welfare increase compared to the *LCD*-solution.

In this way it is possible to both tailor the uniform solutions against optimality and fairness criterions, under the restrictions that the requirement of uniform reductions puts on the solutions. The severity of this constraint depends on which type of *IEP* has to be addressed and in the end determines the relative attractiveness of uniform solutions.

7. Perspectives, policy-implications and discussions

This paper corrects one of the misunderstandings about uniform reductions obligations that have given these types of solutions a bad reputation and the purpose of this analysis has been to shed new light on a set of solutions, that have been political feasible but there merits doubted by economists. Once established that uniform solutions always exist satisfying the individual rationality constraint, it is more easy to comprehend why the use of uniform solutions have been the default solution in many *IEP*, especially in the initial phase of building up a reduction regime, where the privacy of information is high. In such cases, uniform solutions, and especially the LCD-solution, might constitute a unique feasible alternative. Grubb (1996) notes that: “In the case of global warming, raising the question of differentiation among countries potentially opens a Pandora’s box of special leading. The economic systems that emit greenhouse gasses (GHG) are extremely complex. If differentiation is taken to mean trying to negotiate different numerical targets for each individual country according to these circumstances, the scope for never-ending distributional arguments could sink the negotiations (p.47)”. The essence of Grubb’s message is that when countries are much dislike, agreeing on differentiated solutions is a complicated, or even dangerous task, especially as long as information is limited. The robustness of uniform solutions here outweighs the efficiency loss.[17]

Compared to e.g. Hoel (1992) and Barrett (1991), this paper shows that uniform solutions indeed have certain desirable properties, and looking at it more broadly, the choice between uniform and differentiated solutions seems no longer as obvious, but rather context dependent. In the end, the choice amongst them depends on numerous factors like the present state of the information-structure, the set of political restrictions and factors related to the negotiation process. The political restrictions are (to some extent) defined by the characteristics of the IEP. For example, certain characteristics point to the use of uniform reductions, especially when the simplicity of these solutions is essential. This

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[17] It has been noted that uniform solutions are especially important in early stages of the negotiations process, because they are not easily manipulable (see e.g. Brandt 2001).

might be the case when many countries from different parts of the world are involved, or when damages are so high that cost considerations are less important.

The Kyoto agreement has been denoted a successful route. Obviously, the recent breakdown in the climate change negotiations at The Hague indicates the contrary: However, the original Kyoto-agreement stipulated differentiated reduction obligations amounts the different OECD. Moreover, the differentiation is even more severe when taking into account the different business-as-usual growth rates amount the countries.[18] So once again, a small-scale uniform reduction might have been preferable.[19]

Although one cannot simply compare different IEP, there is a clear tendency to move from uniform to differentiated reduction regimes can be observed for many problems. Greene (1996) notes that experiences from the agreements to combat acid rain may provide some of the most relevant lessons on ways of developing differentiated commitments over time. It indicates that it should be more possible to agree on complex differentiated commitments as the first agreement(s) become well established and hence, detailed common understandings have developed of the problem and the responses, and simple, equal commitments have been both agreed and begun to be effectively implemented[20]. In this way, an agreement that only requires status-quo (or very small reductions), but point to the need for further research and so on, might in cases with high uncertainties be that approach with the highest chances for success in the long run. This amounts to say that by including uniform solutions in the set of feasible alternatives, the long run objectives might be easier to reach, not the contrary.

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[18] For more on this, see Brandt and Svendsen (2001).

[19] The US is now going for a renegotiation of the whole Protocol.

[20] An obvious movement is to go from totally uniform to group uniform solution as analysed in Brandt (2001).

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